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**Демонстрационный вариант по направлению «Финансовая экономика»**
**Профиль «Финансовая экономика»**

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# Sample Olympiad Examination Solutions
# November 2013

**Problem 1 solution**

*(a)* *In a closed economy S=I. Given that* $\bar{Y} = 5500$, *I=0.2*5500-100=1000*

*(b)* *Y=C+I+G is pushed to 5550. S becomes 0.2*5550-100=1010. As S=I, then interest rate rises from 50 to 51. The price level rises from 10 to* $11\frac{1}{9}$. *Output expanded by 0.91% while prices increased by 11.(1)%. Since real wage is determined by the marginal product of labour, it should increase proportionally to the change in output, i.e by 0.91%. However, the real wage will decrease by 11.(1)%-0.91%=-10.2% (by Fisher equation) since prices increased.*

*(c)* *Solving* $\frac{M}{10} = 0.2 * 5550 - 20 * 51$ *leads to M=900, i.e. the government should cut the money supply by 100.*

*(d)* *This is due to the fact that either investment or private consumption will fall in order to keep output at the full-employment level. For crowding out effect, this means that investment falls as nominal interest rate rises (less projects remain NPV-positive under higher interest rates). The increase in the nominal interest rate is due to more money injected in the economy to finance government purchases.*

**Problem 2 solution**

Player's 1 view of the game is

| P1\P2 | ℓ | r |
|---|---|---|
| x | $\underline{2}$ , 4 | 1 , $\underline{8}$ |
| y | 1 , 2 | $\underline{2}$ , $\underline{4.2}$ |

In this bi-matrix strategy ℓ is strictly dominated for player 2 so player 1 knows that player 2 will not pick it. Equilibrium (Iterated Dominant Equilibrium) is (y,r). That is, player 1 will play y.

Player's 2 view of the game is

| P1\P2 | ℓ | r |
|---|---|---|
| x | $\underline{2}$ , $\underline{4}$ | 1 , 2 |
| y | 1 , $\underline{2}$ | $\underline{2}$ , 1.8 |

In this bi-matrix strategy r is strictly dominated for player 2 so he will not pick it. Equilibrium (Iterated Dominant Equilibrium) is (x, ℓ). That is, player 2 will play ℓ.

Therefore, the outcome of the game is (y, ℓ). Obviously the outcome is not efficient in the sense that in every player's view there are better outcomes. Moreover, the outcome in not even a Nash Equilibrium from any player's view of the game.

For (b) it is clear that the equilibrium concept is the IDE, which is a stronger concept than the NE.

**Problem 3 solution**

(a) *The first order condition (FOCs) are:* $\begin{cases} \frac{\partial F}{\partial x} = 12x^2 + 20x + 2y^2 = 0 \\ \frac{\partial F}{\partial y} = 4y + 4xy = 0 \end{cases}$*. The second FOC suggests* $y = 0$ *or* $x = -1$*. If* $y = 0$*, then* $x = 0$ *or* $x = -1\frac{2}{3}$*. If* $x = -1$*, then* $y = 2$ *or* $y = -2$*. The critical points are (0,0),* $(-1\frac{2}{3}, 0)$*, (-1,-2), (-1,2).*

*The Hessian is* $\begin{pmatrix} 24x + 20 & 4y \\ 4y & 4 + 4x \end{pmatrix}$*. Evaluate the definiteness of the Hessian for each critical point. For (0,0): positive definite; for* $(-1\frac{2}{3}, 0)$*: indefinite; for (-1,-2): indefinite; for (-1,2): indefinite. Thus (0,0) is the only extrema – a local minimum.*

(b) *The Lagrangean is* $L = 9 - x^2 - y^2 - \lambda(a + bx + cy)$*. The FOC is* $\begin{cases} \frac{\partial L}{\partial x} = -2x - \lambda b = 0 \\ \frac{\partial L}{\partial y} = -2y - \lambda c = 0 \\ \frac{\partial L}{\partial \lambda} = -(a + bx + cy) = 0 \end{cases}$*, which yields* $x^* = \frac{-ab}{b^2+c^2}$ *and* $y^* = \frac{-ac}{b^2+c^2}$*. The bordered Hessian is* $\begin{pmatrix} 0 & -b & -c \\ -b & -2 & 0 \\ -c & 0 & -2 \end{pmatrix}$*. The determinant of this bordered Hessian is* $-2b^2 - 2c^2$*, which is always negative given that* $a \neq 0$ *and* $b \neq 0$*. Hence* $\left(\frac{-ab}{b^2+c^2}, \frac{-ac}{b^2+c^2}\right)$ *is always a local maximum. No additional constraints need to be imposed to guarantee a non-empty set of local extrema of F(x,y).*

(c) *The differential equation can be re-written as* $\frac{y'(x^3 \sin(y) - 3y^2 - 5y^4 x) - y^5 - 3x^2 \cos(y)}{x^3 \sin(y) - 3y^2 - 5y^4 x} = 0$*, which can be further transformed to* $(x^3 \sin(y)\, y' - 3x^2 \cos(y)) - (5y^4 x y' + y^5) - 3y' y^2 = 0$*. It can be seen that this is equivalent to* $(-x^3 \cos(y))' - (xy^5)' - (y^3)' = 0$*. Due to linearity of the differential operator,* $(-x^3 \cos(y) - xy^5 - y^3)' = 0$*. Hence the general solution is* $-x^3 \cos(y) - xy^5 - y^3 = C$*, where* $C$ *is a random constant. Substituting* $y(1) = 0$ *yields* $C = -1$ *and therefore the particular solution is* $x^3 \cos(y) + xy^5 + y^3 = 1$*.*

**Problem 4 solution**

From the first two relations, we obtain

$$\frac{1}{(1+r)^T} = \frac{(C_1 - P_1) - (C_2 - P_2)}{K_2 - K_1} = 0.975,$$

$$S = (C_1 - P_1)\frac{K_2}{K_2 - K_1} + (C_2 - P_2)\frac{K_1}{K_1 - K_2} = 1347.35.$$

Substituting into the third equation, we observe that the put-call parity relation takes the form

$$(C_3 - P_3) + (C_1 - P_1)\frac{K_3 - K_2}{K_2 - K_1} - (C_2 - P_2)\frac{K_3 - K_1}{K_2 - K_1} = 0.$$

Making use of the data given in the table, we obtain a relative mismatch

$$(C_3 - P_3) + (C_1 - P_1)\frac{K_3 - K_2}{K_2 - K_1} - (C_2 - P_2)\frac{K_3 - K_1}{K_2 - K_1} = \$36.68 > 0,$$

which immediately implies the following possible arbitrage strategy that consists of three steps:

1. Sell 1 contract of $C_3$ and buy 1 contract of $P_3$,
2. Sell $\frac{K_3 - K_2}{K_2 - K_1}$ contracts of $C_1$ and buy $\frac{K_3 - K_2}{K_2 - K_1}$ contracts of $P_1$,
3. Buy $\frac{K_3 - K_1}{K_2 - K_1}$ contracts of $C_2$ and sell $\frac{K_3 - K_1}{K_2 - K_1}$ contracts of $P_2$.

**Problem 5 solution**

*(a)* *Marginal costs are $2l + 3r = 9$ for technology A and $1l + 3r = 6$ for technology B. Find the maximum profits for each technology. Let MC=c. $\max_Q((30 - Q)Q - cQ - F)$ yields $Q^* = 15 - 0.5c$. The maximum profit is $(15 - 0.5c)^2 - F$. For A: $c = 9$ and $F = 0$, thus profit is 110.25. For B: c=6 and F=40, thus profit is 104. The firm should use technology A. The price of the output will be 19.5, the quantity produced will be 10.5.*

(b) *There are three stages: technology decision, entry decision and actual production. Use backward induction. For each technology choice there are two sub-games: if firm Z stays out and if it enters. If firm Z stays out, firm N behaves as a monopolist with the chosen technology, making a profit of 110.25 if technology A was chosen or 104 of technology B was chosen (as computed in (a)). Staying out leaves Z with zero payoff. If firm Z enters, find the equilibrium profits for each firm by solving $\begin{cases} \pi_N = (30 - q_N - q_Z)q_N - cq_N - F \rightarrow \max_{q_N \geq 0} \\ \pi_Z = (30 - q_N - q_Z)q_Z - 6q_Z - 70 \rightarrow \max_{q_Z \geq 0} \end{cases}$. This produces $q_N^* = 12 - \frac{2c}{3}$ and $q_Z^* = 6 + \frac{c}{3}$. Firm N's profit is $\left(12 - \frac{2c}{3}\right)^2 - F$, firm Z's profit is $\left(6 + \frac{c}{3}\right)^2 - 70$. Check the profits for both technologies. For A: firm N earns 36 and firm Z earns 11; for B: firm A earns 24 and firm Z makes a loss of 6. Obviously, firm Z will enter if firm N chose technology A and firm Z will stay out if firm N chose technology B (to avoid losses). Thus firm N will anticipates to make 36 if it chooses technology A and 104 if it chooses technology B. Obviously, firm N will choose the latter option (B).*

(c) *In (b) the equilibrium price is 18 and the quantity produces is 12. The quantity is larger and the price is lower compared to A. The ex-post total surplus is 216 in (b) and 165.375 in (a). The ex-ante total surplus is 176 in (b) and 165.375 in (a), which ensures that potential competition in (b) improves social welfare.*