Олимпиада для студентов и выпускников вузов – 2014 г.

Олимпиадное задание по направлению «Психология»

Профиль:

«Измерение в психологии и образовании» КОД - 140

Время выполнения задания – 180 мин.

Блок 1. «Работа с оригинальной статьей, описывающей эмпирическое исследование в области психологии или образования»

Вам предложена статья:

Bassok, M., & Holyoak, K. J. (1989). Interdomain transfer between isomorphic topics in algebra and physics. Journal of Experimental Psychology: Learning, Memory, and Cognition, 15(1), 153

Прочитайте статью и ответьте на вопросы к ней на русском языке.

Вопросы к статье:

В вопросах № 1-6 выберите правильный ответ (или ответы):

1. Какие условия, по мнению авторов, будут помогать успешному переносу знаний из одной ситуации в другую? Поставьте ДА – если условие однозначно улучшает перенос, НЕТ – если однозначно затрудняет, ДН – если нет однозначной связи условия и улучшения переноса.

   1) Внешнее сходство двух ситуаций: ______;
   2) Структурное сходство двух ситуаций: _______;
   3) Сходство контекста двух ситуаций: ________.

2. Какую цель авторы преследуют в первом эксперименте? Поставьте ДА – если указанная цель преследовалась, НЕТ – если цель не преследовалась.

   1) Доказать, что в определённых условиях обучения возможен перенос знания между гуманитарными и естественно-научными дисциплинами: _______;
   2) Доказать, что процесс переноса знаний является асимметричным:_______;
   3) Перенос знаний из области алгебры в область физики может быть достигнут и без большого количества примеров, включающих в себя разный контекст: ________;
   4) Связанность физики со специфическим контекстом будет затруднять перенос знаний из области физики в область алгебры: _______.

3. Что было зависимым показателем в первом эксперименте? Отметьте один ВЕРНЫЙ ответ:

   1) Будет ли применен изученный метод к сходной проблеме, но взятой из другого контекста?

2) Обнаружат ли свое знакомство с проблемой из области физики участники, обученные в области алгебры?
3) Число правильных ответов при решении задания на перенос;
4) Число правильных ответов в претесте, посттесте и задаче на перенос.

4. Какие ранее сделанные выводы подтвердил первый эксперимент? Отметьте один ВЕРНЫЙ ответ:
   1) Разнообразие примеров, используемых в обучении алгебре, ослабляет перенос изученного материала в область физики;
   2) Разнообразие примеров, используемых в обучении алгебре, улучшает перенос изученного материала в область физики;
   3) Интенсивное обучение решению алгебраических проблем улучает перенос в решении структурно схожих физических задач;
   4) Интенсивное обучение решению алгебраических проблем улучает перенос в область физики.

5. Отсутствие переноса из области физики в область алгебры во <u>втором</u> эксперименте авторы объясняют… (Поставьте ДА – если утверждение объясняет результаты, НЕТ – если не объясняет)
   1) …трудностью самих физических задач, отобранных для обучения: _______;
   2) …использованием одного и тоже содержания в тренировочных заданиях по физике: ________;
   3) …привязкой физических уравнений к специфическому контекст: _________;
   4) …тем, что само содержание задач по физике воспринималось как граница их применимости: _________.

6. Какая альтернативная гипотеза проверялась в третьем эксперименте? Поставьте ДА – если указанная гипотеза проверялась, НЕТ - если не проверялась.
   1) Изменение контекста, в котором будет представлена проблема из физики, повлияет на то, как будет осуществляться перенос знаний из алгебры в физику: _____;
   2) Увеличение трудности тренировочных заданий по физике ослабит последующий перенос в область алгебры: ________;
   3) Сложность переноса знаний из физики связана с содержательной спецификой уравнений из физики: ______.

В вопросах № 7-10 дайте развернутый ответ:

7. Какие темы в алгебре и физике были взяты как изоморфные и какие предложены основания считать их таковыми?
8. Какие альтернативные объяснения результатов первого эксперимента возможны, и как авторы их проверяли?
9. Какие цели ставились авторами во втором эксперименте?
10. Какие экспериментальные условия использовались в <u>третьем</u> эксперименте?

Блок 2. «Работа с тезисами эмпирических исследований»
Пожалуйста, прочтите краткое описание каждого из исследований и дайте аргументированные ответы на приведенные ниже вопросы.

Тезисы №1 «Intelligence and Strangeness»
Physicists use a number of constructs such as angular momentum, spin, baryon number, and parity in describing elementary particles. Our favorite construct in physics, however, is that of strangeness. Elementary particles are assigned strangeness numbers that occasionally follow the law of conservation of strangeness, depending upon the type of interaction in which the particles are engaged. We believe that the adoption of this construct in the public schools should be encouraged. Elementary children could be given numbers that would allow teachers to predict the nature of interactions between them. It may be that many of you believe that the IQ score already serves a function similar to that which strangeness serves in physics.
One well-known longitudinal study of this belief was started in 1921 by Lewis Terman. The youngest student in each class (who was relatively likely to have skipped a grade), along with the three brightest (as rated by the teacher), constituted a population that was then tested. The final sample comprised 1528 children with IQ scores above 140—above the ninety-eighth percentile by today's standards. These children were larger, healthier, socially superior, and generally wonderful. As adults this group made remarkable achievements in any number of areas. They also continued to display exceptionally good health, both mental and otherwise.
These data have been used extensively to show that gifted children are not strange creatures who sit in corners and read until their eyes are ruined, but instead are creating a meritocracy. Do you still believe that bright children are a little strange?

Вопросы:

1. Доказывают ли результаты проведённого лонгитюдного исследования то, что IQ можно считать показателем одарённости?
2. Если Вы обнаружили недостатки в планировании исследования, опишите, в чём они состоят, а также, как Вы можете предложить изменить схему исследования, чтобы избежать этих недостатков?

Тезисы №2 «Acting Like an "A" Student
Several years ago, an author by the name of Nelson N. Foote suggested that an individual's socially defined identity may explain why the individual is or is not motivated to perform a task. In a different arena, a social scientist named Stanley Milgram has collected evidence to show that people tend to comply with an authority figure's request, even if the request involves doing something that normally wouldn't be done. Putting these two thoughts together, a researcher recently hypothesized that students would be motivated to excel on a test if they were told by an authority figure to assume the role of someone who is typically thought of as having a highly motivated social identity.
The subjects were 39 college freshmen enrolled in an introductory social psychology class. The experiment was conducted as an in-class activity, with mandatory participation. The procedures were as follows:

> *Subjects were randomly assigned to an experimental (n = 19) and a control group (n=20). The former was read the following instructions: "This is an experiment to determine the effect of a lecture that is read. At the end of the lecture there will be a test. This test will not in any way affect your grade. The lecture is on youth and society;*

> *please try to listen as if you were an 'A' student." The control group was given the exact same directions excluding, "... as if you were an 'A' student." After these directions were completed, subjects were read, in a monotone voice, an intentionally dull lecture lasting approximately 15 minutes. At the completion of the lecture, a brief multiple choice exam was administered.*

After the multiple-choice exam had been scored, the resulting data were subjected to a formal statistical test. This statistical comparison verified the research hypothesis, for the experimental group achieved significantly higher scores than the controls. The researcher ended the published paper with this statement: "Since both groups were asked specifically to listen to the directions, but only the experimental group was asked to assume a motivated social identity, it is concluded that these data suggest support for Foote's (1951) theory as well as extending Milgram's (1963) work on compliance."

A cause-and-effect relationship was investigated and allegedly identified in this experiment with the 39 college freshmen. The effect variable was the earned score on the multiple-choice exam, and the causal variable presumably was the presence or absence of the phrase "as if you were an 'A' student" at the end of the directions. Can you think of any alternative explanation(s) to account for the significant difference between the performances of the two groups on the exam other than this slight difference in what the subjects were told prior to the dull lecture? Or do you accept the conclusions as valid?

Вопросы:

3. Какой информации в описании эксперимента Стенли Милгрэма не хватает для того, чтобы согласиться или опровергнуть достоверность вывода?
4. Приведите, пожалуйста, не менее трёх альтернативных объяснений полученному результату.

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Interdomain Transfer Between Isomorphic Topics in Algebra and Physics

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Three experiments examined transfer between two isomorphic subdomains of algebra and physics. The two areas were arithmetic-progression problems in algebra and constant-acceleration problems in physics. High school and college students who had learned one of these subtopics were presented with word problems that used either content from the domain they had originally studied or content based on the unfamiliar but analogous domain. Students who had learned arithmetic progressions were very likely to spontaneously recognize that physics problems involving velocity and distance can be addressed using the same equations. Analysis of problem-solving protocols revealed that the recognition was immediate and that the solutions were a straightforward application of the algebraic method. Such recognition occurred even when the algebraic procedures were taught using example word problems all of which were drawn from a single content area (e.g., "money" problems). In contrast, students who had learned the physics topic almost never exhibited any detectable transfer to the isomorphic algebra problems. In the only case of transfer from physics to algebra, the process was analogical in nature. In addition, transfer from algebra to physics word problems was impaired if the physics transfer problems were embedded in a discussion of motion concepts. The results were interpreted in terms of content-free versus content-specific applicability conditions for mathematical procedures.

One of the most persistent results in studies of human problem solving is that experience with particular problems often yields little or no transfer to similar problems. This negative conclusion emerges from studies of transfer between isomorphs of the "Tower of Hanoi" problem (Hayes & Simon, 1977), between homomorphs of the "missionaries and cannibals" problem (Reed, Ernst, & Banerji, 1974), and between slightly transformed versions of algebra problems (Reed, Dempster, & Ettinger, 1985). On the surface, the generally dismal experimental findings regarding transfer in problem solving would seem to undermine the most fundamental goals that guide education. Except for the simplest forms of rote learning, effective instruction is intended to impart knowledge that can be applied to situations other than those that were directly taught. Indeed, because it is often impossible to foresee precisely the situations in which particular knowledge will prove potentially useful for the student, educators are often concerned to teach in such a manner that transfer to novel problem situations will be possible.

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Determinants of Transfer

The major requirement for ensuring successful transfer is to foster the problem solver's access to relevant prior knowledge. Teachers do, of course, often directly instruct students that certain domains are related, thus eliminating the problem of spontaneous access. For example, Gentner and Gentner (1983) found that the analogy between electric circuits and water flow, which is often directly used in instruction, systematically influenced subjects' solutions of electricity problems. In some studies (Reed et al., 1974; Gick & Holyoak, 1980, 1983), an explicit hint to use an initial problem as a guide to help solve an analogous problem from a different domain increased the level of transfer. Without such external prompts, prior knowledge often remains "inert" (Whitehead, 1929).

Access to knowledge will be highly dependent on the way in which it is organized in memory. In particular, if we accept the notion that problem-solving knowledge is typically encoded along with its conditions of applicability (Anderson, 1983; Bransford, Sherwood, Vye, & Rieser, 1986; Cheng, Holyoak, Nisbett, & Oliver, 1986; Simon, 1980), it follows that access to knowledge will depend on the relation between the encoded conditions of applicability and the retrieval cues afforded by the transfer task. It is useful to distinguish three general classes of information that may be used to define conditions of applicability: surface content, underlying structure, and context.

Content cues include salient features of the specific domain from which examples are drawn, whereas structural cues are less salient features, often relational in nature, that directly relate to the conditions under which particular solution methods are in fact appropriate. Holyoak and Koh (1987) found that both content and structural cues affect access to earlier examples of problems. Content cues are very likely to trigger reminding (Ross, 1984, 1987); however, to the extent that content cues are not perfectly correlated with underlying structural features, reliance on the former may lead to negative transfer across problems that are only superficially similar and to failure to access related knowledge across different content domains. Gick and Holyoak (1983; also Brown, Kane, & Echols, 1986) found that exposure to multiple examples with differing contents helped subjects to focus on shared structural features, which in turn facilitated transfer.

Contextual cues derive from the situation in which the initial information is encoded; if the context of the transfer task differs substantially, transfer will be impaired (Spencer & Weisberg, 1986). It is important to note that context is not limited to the physical components of the situation. The psychological context, such as the set of expectations people have about the problem-solving task in which they are engaged, will influence access to knowledge. Schoenfeld (1985), analyzing the components of mathematical knowledge and behavior, referred to such psychological expectations as "belief systems." He found that students could be quite capable of using deductive argumentation while solving geometry "proof problems," yet fail to invoke their deductive knowledge when solving "construction problems," for which they believed the appropriate approach was trial and error. In such a broad sense, context may have a major influence on the choice of content and structural cues for inclusion in applicability conditions.

It is often the case that contextual, content, and structural features are naturally confounded in everyday problem solving. In the case of physics, for example, the structural features crucial to the applicability of physical laws are correlated with specific physical concepts and are learned within the school context of studying physics. As a result, physics knowledge is often learned in a manner that allows it to be applied to textbook situations, but not to everyday physical phenomena encountered outside of the school context (see, e.g., Caramazza, McCloskey, & Green, 1981; di Sessa, 1982; McCloskey, 1983).

There are some topics, however, in which the three types of cues are not correlated to the same degree. Mathematics and logic, for example, are conceived as tools that are essentially content free and independent of any particular domain of application. To the extent that students in fact acquire domain-independent representations of formal procedures, coupled with suitably abstract conditions of applicability, such knowledge might be applied appropriately to novel content areas.

There is relatively little empirical evidence concerning transfer of formal problem-solving procedures. Cheng et al. (1986) found that purely formal training in propositional logic had little impact on subjects' ability to solve inference problems of an unfamiliar type. However, if logic training was coupled with a small number of example problems, subjects were in fact able to apply the logical procedures to isomorphic problems with different content. Fong, Krantz, and Nisbett (1986) found that instruction in basic statistical principles yielded transfer to problems with different surface features.

In the case of mathematics, Reed et al. (1985) found that novice students of algebra were unable to solve even a slightly transformed version of an example problem. Their negative findings may, however, be limited to students with minimal initial learning. Within the domain of mathematics, as well as within the domain of physics, there is considerable evidence that people do learn fairly abstract categories of problem situations, which might be expected to facilitate transfer. Chi, Feltovich, and Glaser (1981) examined the ways in which novices and experts categorized physics problems. Although experts categorized problems more abstractly than did novices, and in ways more conducive to finding efficient solutions, even novices were able to recognize that a fairly broad range of problems shared the same schematic structure (also see Larkin, McDermott, Simon, & Simon, 1980). Other studies have shown that students are able to reliably categorize different types of algebra word problems they have encountered in the school curriculum (Hinsley, Hayes, & Simon, 1977; Mayer, 1981). The mode of text comprehension and problem solving for algebra word problems proposed by Kintsch and Greeno (1985) assumes that problem solvers engage in an understanding process that depends on retrieval of problem schemas. To some extent at least, students appear to have learned how to translate problems with a variety of "cover stories" into formulas appropriate to the underlying relational structure. These studies of problem categorization did not, however, provide any direct evidence linking the ability to classify problems within each domain to successful transfer.

Isomorphic Subtopics of Algebra and Physics

Our goal in the present study was to investigate problem-solving transfer based on the ability to apply mathematical procedures to simple word problems in algebra and physics. The study centered on two specific subtopics of the high school algebra and physics curriculum, namely, arithmetic-progression word problems (algebra), and problems involving motion in a straight line with constant acceleration (physics). These two domains have several advantages for a study of transfer. Each has a rigorous formal structure, and each affords a wide range of examples dealing with a variety of everyday content domains. In addition, dealing with problems from a typical high school curriculum has the benefit of possible direct implications for the educational system with regard to optimal structuring of the school curriculum.

Most important, the word problems within these two domains, although referring to very different contents, are structurally isomorphic. In general terms, an arithmetic-progression problem involves an expanding numerical sequence that begins from some initial term and increases in increments of a constant magnitude (the *common difference*) up to some final term. The basic questions that can be asked about an arithmetic progression are the following: (a) How many *terms* are there? (b) What is the *value* of the *first term*? (c) What is the *value* of the *final term*? (d) What is the *common difference*? (e) What is the *total sum* of these terms?

The italicized words refer to the major abstract categories involved in defining a schema for arithmetic-progression problems. Students are typically taught these terms, as well as equations sufficient to solve the kinds of problems that result from the above questions.

The content of the isomorphic physics problems refers to moving objects that increase in velocity at a constant rate while moving in a straight line. Typical types of questions, corresponding to the algebra questions presented above, are the following: (a) How many *seconds* did the body *travel*? (b) What was the *velocity* at the beginning of the *first second*? (c) What was the *velocity* at the end of the *final second*? (d) What is the *constant acceleration*? (e) What was the *total distance* traveled?

The isomorphism depends on mapping initial velocity onto the initial term in a sequence, discrete units of time onto the number of terms in the sequence, constant acceleration onto the common difference, and final velocity onto the final term. Total distance maps onto the sum of a different arithmetic sequence, in which each term is the distance traveled in the corresponding unit of time.

Each of these topics is commonly taught in Grade 11, but in different classes. Furthermore, despite the isomorphism that exists between the two types of problems, neither teachers nor textbooks typically mention that the topics are related, although some algebra textbooks may include one or two word problems referring to motion. Our analysis of content specificity of representations, however, allows us to make predictions about the degree of transfer we would expect between each topic and the other.

In the case of arithmetic progressions, the equations and associated terminology for variable assignment are very abstract and are intentionally taught so as to fit a wide range of contents given a certain set of necessary conditions. The relations between the various variables are illustrated abstractly by exercising with problems without any specific content, and the variables in the relevant equations are nonmnemonic. The surface features illustrated in typical word problems vary widely and include such differing contents as whether the story deals with deposits to a saving account, with lengths of metal rods, or with the number of seats in an auditorium. The problems also include other semantic elements that students learn are generally irrelevant to the solutions of algebra word problems (Kintsch & Greeno, 1985). A simple rule for deciding that the equations might be applicable would be "If a word problem is presented, and it mentions a constantly increasing quantity, then try the arithmetic-progression equations." Such a content-free rule would allow even a novice to apply the equations to word problems with novel content. Accordingly, we predicted that procedures for solving the algebra problems should be applicable to physics problems, even if the content of the latter is completely unfamiliar, resulting in a high level of transfer. Since constant-acceleration problems are in fact a special case of arithmetic progressions, transfer from algebra to physics can be viewed as a case of categorizing novel instances (constant-acceleration problems) as members of a known category (arithmetic-progression problems).

In contrast, studying physics would be expected to result in representations that are bound to the physics content. The formulas are taught as a means of solving a very distinct and specific group of problems, those dealing with a body moving in a straight line with constant acceleration. The variables in equations are given mnemonic names related to physical concepts, such as "acceleration." The critical structural relations, which students are taught to check, involve a moving body that travels in a straight line, the speed of which increases or decreases by a constant amount every successive unit of time over some time interval. A simple rule for identifying word problems of this type would be "If a physics word problem is presented, and it involves time, and speed or distance, and constant acceleration, then try the constant-acceleration equations." The resulting representation is certainly abstract to a degree, in that the student learns to ignore various kinds of surface information, such as the nature of the moving body (a car, train, ball, etc.) and such details as the purpose of the trip. Nonetheless, it is clear that the physics representation will be more content bound than the corresponding algebra representation.

One illustration of the difference in content specificity is the fact that the parameters in arithmetic progressions have no units, and the number of terms is a pure count, whereas the parameters of constant-acceleration equations have specific units that bind the meaning of their products. For example, multiplying acceleration (m/s^2) by time (s) results in velocity (m/s), and multiplying velocity (m/s) by time (s) results in distance (m). Thus, whereas the representation for arithmetic-progression problems is very abstract and applicable to virtually any content, the representation for constant-acceleration problems is bound to specific content, units, and concepts. Accordingly, we predicted a low level of transfer from physics to algebra, because transfer in this direction would require a relatively demanding analogical "remind-and-map" process (Holyoak, 1985; Holyoak & Thagard, in press) to relate a relatively domain-specific representation (for constant-acceleration problems) to analogous problems in other content domains.

Bassok and Holyoak (1985) performed an initial pilot test of these predictions in a study conducted within a natural school setting. The study involved 11th-grade high school subjects from an algebra class who had studied arithmetic-progression problems but not constant-acceleration problems and 11th-grade high school subjects from a physics class for whom the reverse was true. A striking asymmetry was observed in transfer performance. Students who had learned the arithmetic-progression method applied it equally often (72% of the time) to new algebra problems (with content similar to that of the problems used in training) and to novel physics-content problems. In contrast, students who had learned the constant-acceleration method applied it to all of the physics transfer problems but to none of the algebra transfer problems. This initial study thus strongly supported our predictions regarding the impact of differential content specificity on transfer of mathematical procedures. However, given the methodological limitations of conducting studies in the school setting, a more rigorous replication seemed desirable.

The present study had several aims. Experiment 1 was designed to demonstrate that interdomain transfer of mathematical procedures is indeed possible under the optimal conditions of algebra training. In addition, we sought to confirm that transfer is indeed asymmetrical, in that the domain specificity of physics instruction would make transfer to nonphysics problems very difficult. We also wished to test alternative hypotheses regarding the necessary conditions for transfer of algebraic knowledge. Experiment 2 investigated transfer when the range of training examples was restricted, and Experiment 3 explored the effects of content specificity of the context during the transfer task.

Experiment 1

Experiment 1 was essentially a replication of the initial experiment by Bassok and Holyoak (1985), except that the students were instructed individually, gave "talk aloud" protocols while solving both the base domain and the target problems, and solved a set of pretest problems structurally matched to the final set of problems. The individual instruction enabled us to control the exact content and presentation of the material to be learned, and the verbal protocols provided us with much more detailed information about the solution process. The solutions of the pretest problems provided a baseline for comparison between the methods applied before and after studying the base domain subject matter.

Method

Subjects. Subjects were 12 ninth-grade high-ability students from an accelerated scientific program in a public high school in Pittsburgh. They had not yet studied either the topic of arithmetic progressions or the topic of motion in a straight line with constant acceleration. There were 5 male and 7 female subjects. Subjects were paid for participating in the study.

Materials. Two sets of algebra problems and two sets of physics problems were constructed. One set of arithmetic-progression problems and one set of constant-acceleration problems served as pretests for the algebra and the physics groups, respectively. The other two sets served as transfer test and posttest for each of the groups. The algebra and physics problems were matched in pairs with respect to the underlying structure of the problem. The first two problems in each set were of the arithmetic-sequence type, and the third was an arithmetic-series problem. Each pair could be solved using a corresponding formula and was matched with respect to which variables were given and unknown. Examples of the word problems used are presented in Table 1.

Two shortened and somewhat revised versions of standard texts used currently in high school were used to teach the relevant physics and algebra subject matter (the original texts are by Murphy, Hollon, & Zitzewitz, 1982, [pp. 39–47] and Dolciani, Wooton, Beckenbach, & Sharon, 1983 [pp. 213–255]). The revised version of the algebra chapter omitted the section dealing with the Σ, and presented only a subset of the original practice problems. The revised version of the physics chapter excluded all references to changing units (for example, changing from meters per seconds into feet per minutes), vectors, velocity as distinct from speed, and references to other chapters.

Table 1
Matched Arithmetic-Progression (Algebra) and Constant-Acceleration (Physics) Test Problems: A Representative Set

Algebra	Physics
Sequence type Given: a_1, d, n, find a_n	
1. A boy was given an allowance of 50 cents a week beginning on his sixth birthday. On each birthday following this, the weekly allowance was increased 25 cents. What is the weekly allowance for the year beginning on his 15th birthday?	1. An express train traveling at 30 meters per second (30 m/s) at the beginning of the 3rd second of its travel, uniformly accelerates increasing in speed 5 m/s each successive second. What is its final speed at the end of the 9th second?
Given: a_1, a_n, n, find d	
2. During a laboratory observation period it is found that the diameter of a tree increases the same amount each month. If the diameter was 8 mm at the beginning of the first month, and 56 mm at the end of the 24th month, by how much does the diameter increase each month?	2. What is the acceleration (= increase in speed each second) of a racing car if its speed increased uniformly from 44 meters per second (44 m/s) at the beginning of the first second, to 55 m/s at the end of the 11th second?
Series type Given: a_1, a_n, n, find S_n	
3. A mechanic has to cut 9 different length metal rods. The shortest rod has to be 6 ft. long and the longest rod has to be 10 ft. long, and each rod has to be longer than the one before by a constant amount. What is the total length of metal required to prepare these rods?	3. A jumbo jet starts from rest and accelerates uniformly during 8 seconds for takeoff. If it travels 25 meters during the first second and 375 meters during the 8th second, what distance does it travel in all?
Given: a_1, d_1, n, find S_n	
4. Kate O'Hara has a job that pays $7,500 for the first six months, with a raise of $250 at the end of every six months thereafter. What was her total income after 12 years?	4. An object dropped from a hovering helicopter falls 4.9 meters during the first second of its descent, and during each subsequent second it falls 9.8 meters farther than it fell during the preceding second. If it took the object 10 seconds to reach the ground, how high above the ground was the helicopter hovering?

Procedure. All subjects were recruited for a study described as directed at the development of new instructional materials for high school mathematics and science curricula. All students expected to study both an algebra and a physics chapter. Subjects were randomly assigned to two experimental conditions according to the base domain they had to study. Six subjects were assigned to the physics group. These subjects studied the physics chapter and later were tested on arithmetic-progression problems. The other 6 subjects, the algebra group, studied the algebra chapter and were subsequently tested on constant-acceleration problems.

The experiment was conducted in two experimental sessions. The first "pretest and study" session lasted 1.5–3 hr, and the second "transfer" session lasted between 30–60 min. For each subject, the two sessions were conducted on two consecutive days.

During the first session, subjects solved a set of three pretest problems from the chapter they were about to study, giving "talk aloud" protocols. The physics subjects solved three constant-acceleration problems, whereas the algebra subjects solved three arithmetic-progression problems. The pretest problems for the two groups, like the later test problems, differed only with respect to their content and were matched in pairs with respect to their underlying structure.

After finishing the pretest, subjects were given a chapter covering all the necessary information for the base domain. Students studied at their own pace with minimal intervention from the experimenter, solving some practice problems as they progressed. When they felt that they had mastered the chapter, they were given a test covering the various topics presented in the text. Their test was corrected, and any mistakes were pointed out to the subject, who was asked to reread the chapter until correctly solving all of the test problems. This procedure of learning until criterion was used to increase the probability that all of the subjects would adequately master the methods for solving problems from the base domain.

During the second session, conducted on the following day, subjects were first told they would be learning about a new domain. They were then asked to solve three transfer problems (i.e., problems from the domain that they had not studied). These problems were presented as a pretest to the new chapter they were about to study. In none of the experiments to be reported were subjects informed about the relationship between their training and the transfer test. After finishing the transfer test, subjects were asked to solve the three parallel problems from the studied base domain. Subjects were then informed that they would not in fact study another chapter and were thoroughly debriefed. The entire session was tape-recorded for later analysis.

Results and Discussion

Quantitative analyses. Our main concern was to determine whether subjects were able to recognize that the unfamiliar constant-acceleration problems were amenable to the same type of solution as were the familiar arithmetic-progression problems, because of their shared underlying structure. Accordingly, the primary measure of transfer involved an assessment of the solution method applied to the problems, rather than the final correct answers. We categorized the solution methods exhibited in subjects' written work into two main categories, the "learned" method and "other" methods. A solution attempt was classified as using the learned arithmetic-progression method if the solution included explicit arithmetic-sequences or arithmetic-series formulas ($a_n = a_1 +$ sion notation as taught in the class (a_1, a_n, n, d, S_n). It was classified as using the learned constant-acceleration method if the solution included explicit constant-acceleration formu- acceleration notation as taught in the class (v_i, v_f, t, a, S).

The "other" methods included two subcategories. (a) In the means–ends method, the solution attempt included solutions of subproblems and involved application of general algebraic equations for the solution of the subproblems (e.g., multiplying the number of required additions by the size of the common difference and then adding the resulting value to the first term). This method captures the rationale behind the basic formula for arithmetic sequences, but it uses more steps and does not explicitly use the specific notation. (b) In the one-by-one method, the solution attempt consisted of sequential additions iterated until the requested amount was reached, often after a trial and error search for the magnitude of the common difference. Such a method becomes very cumbersome and time consuming as the number of terms in the sequence or series increases. Because no interesting patterns emerged from the analysis of subtypes of "other" methods, only the frequencies of arithmetic-progression solutions are reported here.

Table 2 summarizes the various quantitative performance measures for subjects from both experimental conditions on the three parallel sets of pretest, posttest, and transfer problems. The left column presents the results concerning the solution method, the middle column presents the average number of correct solutions, and the right column presents the solution time.

The main dependent measure of transfer in the present study was whether the learned method had been applied to structurally isomorphic but unfamiliar problems. Because none of the subjects were initially familiar with either the physics equations or with the specific algebraic equations, the methods that were applied for the solution of all of the pretest problems came from general algebraic knowledge available to the subjects. After studying the relevant chapter, all the algebra students applied the learned arithmetic-progression method to the posttest algebra problems, and they applied the method to 17 out of the 18 available problems. Similarly, all the physics students applied the learned constant-acceleration method to the posttest physics problems, and they applied the method to 16 out of the 18 available problems. Thus both groups acquired and used the learned method for the familiar base-domain problems. The two training groups behaved very differently, however, with respect to transfer. Whereas all

Table 2
Performance Measures for Algebra and Physics Problems Under Algebra Training and Under Physics Training in Experiment 1

Training type	% application of learned method	% correct answers	Solution time per problem (minutes)
Algebra training ($N = 6$)			
Pretest (algebra)	0	72	2.35
Posttest (algebra)	94	100	1.45
Transfer (physics)	72	94	1.31
Physics training ($N = 6$)			
Pretest (physics)	0	83	3.48
Posttest (physics)	89	100	1.88
Transfer (algebra)	10	94	2.37

Note. Percentages are based on the average number of problems per subject, out of a total of three problems.

algebra students applied the learned method to the solution of the physics problems (13 out of 18 problems), only one physics student used the learned method for the unfamiliar algebra problems (2 out of 18 problems).

With respect to solution method, the results were thus straightforward. Whereas there was very high transfer from algebra to physics (72%), there was almost no transfer from physics to algebra (10%). This difference is of course highly significant, $F(1, 10) = 25.3$, $p < .001$, $MS_e = .53$, for the interaction between the training and the content of problems.

The number of correct solutions served only as an indirect measure of transfer, because all the problems could have been solved correctly even when solved by means of a different, more cumbersome method. Answers were scored as correct either when the answer was entirely correct or when the error resulted from a calculation slip (including miscalculation of the number of terms in the sequences). Indeed, although the physics subjects did not apply the learned method to the unfamiliar algebra problems, they correctly solved all of the transfer problems, performing as well on these problems as did the algebra subjects. Thus with respect to the number of correct solutions, both groups improved in comparison to their performance on the pretest, $F(1, 10) = 6.45$, $p < .01$, $MS_e = .21$, with no significant interaction between training and test condition. This result suggests that some learning may have occurred in the physics group that was relevant to the solution of the algebra problems. However, improvement was not due to a qualitative change in solution methods. In fact, transfer problems that were not solved by the learned method were solved by the same methods applied to the matching pretest problems, either means–ends or the one-by-one iterative method. The improvement thus reflects the effect of practicing general algebraic methods that the physics subjects knew prior to the experiment.

Analysis of the solution time for the various problems provides additional evidence for an asymmetry between the algebra and the physics training groups. These times were calculated from the tape-recorded protocols, adjusted by subtracting time devoted to explicit mathematical calculations. (The pattern of solution times was the same when unadjusted times were considered.) Although both groups required more time to solve the pretest problems than either the base-domain posttest problems or the transfer problems, $F(1, 20) = 11.3$, $p < .001$, MS_e (in seconds) = 14,637, the physics subjects required more time to solve the algebraic-progression problems than the algebra subjects required to solve the physics problems. This effect of training condition was qualified by significant interactions between test and problem type (sequence vs. series), $F(2, 20) = 9.11$, $p < .001$, and between training condition, the test and the problem type, $F(2, 20) = 6.00$, $p < .01$, $MS_e = 2{,}509$ for both interactions. A more detailed analysis of the solution-time differences revealed that the effect of training condition was entirely due to relatively slow solution of the algebraic-series problem by the physics subjects, who typically applied the cumbersome iterative method (average of 3.89 min/problem). In contrast, the algebra subjects required an average of only 2.07 min to solve the corresponding series problem in physics using the efficient arithmetic-series formula, $t(20) = 3.77$, $p < .001$. The two groups did not differ in solution time for the sequence problems (1.61 and 1.23 min/problem for physics and algebra training, respectively), $t(20) < 1$. (Note that the mean solution times reported in Table 2 are based on a weighted average of the means for the one series problem and the two sequence problems.) Whereas the equation for algebraic series provides a qualitatively different and more efficient solution procedure (calculating the value of the average term and multiplying it by the number of terms), the equation for algebraic sequences is essentially an explicit formulation of the means–ends method that most of the subjects used spontaneously, and hence its use does not produce shorter solution times.

Protocol results. In order to examine the basis of the observed asymmetry in transfer in more detail, the verbal problem-solving protocols of subjects in both the algebra and the physics groups were analyzed. We were particularly interested in identifying the initial signs of recognition that the transfer problems were similar to those of the type used in training.

Table 3 presents three respective excerpts from the protocols of algebra-trained subjects solving physics problems on the transfer test. These subjects referred to the physics problems as a simple new case of the familiar arithmetic-progression problems. They proceeded by retrieving the relevant equation, identifying the relevant variables, and inserting the values into the equation. The most striking feature of the transfer protocols for all the algebra students is the absence of any indication that the subjects are dealing with unfamiliar problems. The verbal reports are entirely consistent with our hypothesis that algebra training results in the formation of generalized rules describing applicability conditions, with abstract variables that can be readily matched by the components of physics problems with the same structure. Transfer is simply the result of applying information about a known category to a new instance.

Table 3
Examples of Protocols in Experiment 1: Transfer From Algebra to Physics

Algebra subject 1, solving physics problem 1
I'm recalling the arithmetic progression
labeling the third second a_1, the fourth second a_2 . . .
The equation was $a_n = a_1 + (n - 1)d$
and d equals 5
now n equals 7
a_7 is what we want to find out
is equal to a_1 which is 30, plus $(n - 1)$ which is 6,
times d which is 5, which is equal to 60.

Algebra subject 2, solving physics problem 1
8 increases
9 − 1 (n-1)
times the *common difference*, which is 5 m/s
and that should be equal to the final speed.

Algebra subject 6, solving physics problem 3
All right
a_1 equals 25, a_n equals 375,
and d equals the acceleration.
and now I have to do $S_n = (a_1 + a_n)\, n/2$.

Note. The problems appear in Table 1.

The protocols provided much less information about the mechanism of transfer from physics to algebra, considering that only one subject noticed the relevance of the physics equations to two of the three arithmetic-progression problems that she received. Nonetheless, her protocols, the crucial portions of which are excerpted in Table 4, are quite revealing. It is readily apparent that the recognition of similarity proceeds in a much more laborious fashion than in algebra-to-physics transfer. The protocols illustrate the indirect remind-and-map process of transfer incorporated in the computational model of analogical problem solving proposed by Holyoak and Thagard (in press).

The subject's first noticing of the analogy came in the second of the three transfer problems, which involved a sequence of diameters of a growing tree (Table 4). The first clue that seems to have triggered the reminding was based on the similarity between constant increase in the diameter of a tree and constant acceleration, both of which involve a constant rate of change over time. Once she had been reminded of the physics equation, the subject tried to map the remaining variables into the relevant slots, mapping initial diameter onto initial velocity, final diameter onto final velocity, and the number of months onto the time slot. The analogical nature of the hypothesized correspondences is suggested by such statements as "the beginning [diameter] . . . it's *like* the beginning speed." It is apparent in the subsequent portion of the protocol that the analogy was very tentative; however, she proceeded to work out the suggested solution, which was in fact correct.

Table 4 also presents excerpts of the same subject's protocol as she solved the third problem, which involved finding the sum of the lengths of a series of rods. Once again the semantic link between a constant increase and acceleration provided the first reminding cue, again followed by an explicit development of the mapping. However, one important difference between this protocol and that for her first use of the analogy concerns the degree of abstraction of the terms she used to refer to variables. For example, whereas in the first protocol she referred to the initial diameter as being "the beginning speed," directly indicating the analogy, in the second protocol she referred to the initial length as "the first amount," a more generalized and domain-independent concept. This suggests that in the process of working out the analogy the first time, the subject induced more abstract concepts that begin to approximate the representation for arithmetic progressions that algebra-trained subjects were taught directly. The theoretical implication is that the indirect remind-and-map process can provide the opportunity for induction of relatively abstract, domain-independent applicability rules, which subsequently allow more direct transfer.

The rest of the physics subjects never even mentioned the possibility that the previously learned physics material might be relevant to the new algebra problems. In the absence of any surface similarities, analogical reminding is a relatively rare event.[1]

Table 4
Examples of Protocols in Experiment 1: Transfer From Physics to Algebra

Physics subject 1, solving algebra problem 2

Umm . . . well . . . I know there is a certain connection.
It's something *like acceleration* . . .
I mean it's really similar, because there is time and . . .

Maybe we can use a formula for acceleration, because
you have the beginning . . . it's *like the beginning speed, the final speed*, and *the time.*

The *8 mm* as the beginning *speed*
and the *56 mm* as the end *speed*
and the 24 is umm . . . time.

Because it has the *same kind of variables.*

But maybe it isn't, we don't have a proof that it is.
I'll just try using that formula, maybe it will work.

It's like . . . for acceleration.
Cause we want to know *what the acceleration is*
Because they're asking us *how much it grew each month.*

Physics subject 1, solving algebra problem 3

So, what we have to find again is the difference between each rod,
because they say it's a constant amount of difference.

Umm the difference between each rod . . . and it's constant.
OK, and that's again *like the acceleration.*

Because you have the *first amount*
and you have a *final amount*
and you have *something like the time.*

Note. The problems appear in Table 1.

Experiment 2

The results of Experiment 1 confirmed the findings of Bassok and Holyoak (1985), indicating that instruction in algebraic problem-solving procedures produces strong transfer to structurally parallel physics problems with novel content. As argued earlier in this article, there are a number of factors that would be expected to make representations of algebraic knowledge relatively independent of content cues. Experiment 2 was designed to provide more detailed evidence regarding the importance of one of the most salient of these factors: the diversity of the examples used in training. In Experiment 1, as well as in the earlier classroom study performed by Bassok and Holyoak (1985), arithmetic progressions were illustrated using word problems drawn from several distinct content areas (e.g., growth of savings accounts, height of a human pyramid). If students formed generalized rules for applicability conditions by an intersection process (Hayes-Roth & McDermott, 1978; Winston, 1975), diversity of examples might be crucial to successful transfer.

It is possible, however, that interdomain transfer of algebraic procedures might be achieved even without diverse examples. After years of extensive training in solving word

[1] One other physics student, in a study not reported here, showed transfer to a single algebra problem. Her protocol also provided clear evidence of the relatively laborious task of developing an analogical mapping.

problems, starting with simple arithmetic problems (e.g., "Joe has 2 marbles and Mary gave him 3 more"), it is possible that algebra students will already have learned that the specific content is typically irrelevant to the applicability of equations and hence will not include content-specific details as applicability conditions. This would be an example of the kind of "belief system" that Schoenfeld (1985) suggested is important in guiding the application of mathematical knowledge.

In Experiment 2 we examined whether instruction in arithmetic progressions would transfer to word problems in a novel domain even if the initial training involved examples with only a single content (e.g., money problems). In addition, Experiment 2 provided a test of whether the lack of transfer from physics to algebra, observed in Experiment 1, could be attributed to some intrinsic difficulty in learning from examples of motion problems. One of the groups in Experiment 2 received instruction in arithmetic progressions coupled with constant-acceleration problems as the sole examples. If there is some intrinsic difficulty in learning from examples of motion problems, this group would show reduced transfer. On the other hand, if the lack of transfer from physics in Experiment 1 was due to the content specificity of the physics equations, rather than to the intrinsic difficulty of learning from motion problems, then such problems would produce as much transfer as any other content domain when used to illustrate algebraic equations.

Methods

Subjects. Subjects were 22 undergraduate college students, 10 men and 12 women. All students attended an Algebra II class at the University of Pittsburgh that was offered to students who did not pass the university's algebra placement test. The subject population was thus considerably older than the ninth-grade subject population used in Experiment 1, but less talented in mathematics. The experiment was conducted during the regular lecture and recitations as a part of the Algebra II course. A total of 89 students were in the course, but only 22 were present for all four experimental sessions. None of the students was familiar with either the topic of arithmetic progressions or with the topic of accelerated motion.

Materials. All subjects received the same materials for instruction in algebra as were used in Experiment 1, except that the examples of worked-out word problems were deleted. In lieu of the latter, three new training sets of word problems were constructed, based on three distinct contents: money, people, and motion. The money problems dealt with salaries, loans, and investments. For example:

> Juanita went to work as a teller in a bank at a salary of $12,400 per year, and received constant yearly increases coming up with a $16,000 salary during her seventh year of work. What was her yearly salary increase?

The people problems dealt with population growth, student enrollment, numbers of people in rows, and so on. For example:

> In 1975 a total of 22,630 people attended conventions at a convention center. If 2,000 more people attend conventions at the center each year, how many people will attend the center in 1990?

Finally, the physics problems dealt with bodies moving in a straight line with constant acceleration. Each set consisted of two examples of worked-out problems and four practice problems. One worked-out problem and two practice problems had the structure of arithmetic sequences, and the other worked-out problem and two practice problems had the structure of arithmetic series. Solutions for the four practice problems were provided after subjects attempted to solve them.

For use in pretest, posttest, and transfer sessions, six parallel sets of word problems were constructed, two sets for each of the three content areas. The first two problems in each set had the structure of arithmetic sequences and the last two problems had the structure of arithmetic series. The sets were equated with respect to which variables were given and which had to be solved for. Note that whereas the problem sets in Experiment 1 included two sequence problems and one series problem, in Experiment 2, we used an additional series problem. This addition was intended to increase the sensitivity of the transfer set. As mentioned above, the learned method for the series problems, unlike that for the sequence problems, was qualitatively different from the methods available to subjects before studying the relevant chapter.

Procedure. The study was conducted in the Algebra II class during regularly scheduled lectures and recitations. Students were informed by their instructor that the following four classes would be taught by a research team from the Learning Research and Development Center affiliated with the University of Pittsburgh, who were developing teaching materials in mathematics and science. Students were asked to attend all the classes. The four experimental sessions were conducted on 4 different days within 1 week. The activities on each day were as follows:

1. *Pretest session*—Students took 20 min at the beginning of their recitation to solve four pretest problems, consisting of one of the sets for a given content. Seven students received money problems, 5 students received people problems, and 10 students received physics problems. The regular teacher then took over to discuss previous homework.

2. *Training in algebraic equations*—All students received a lecture given by the experimenter, which consisted of the arithmetic-progressions chapter as revised for the experiment. Each student received an individual copy of the chapter. Overheads of the chapter were presented by the experimenter and were read out loud with minor elaborations. Students were asked to solve the exercise problems included in the chapter as homework, and to bring them back to their next algebra recitation. This session lasted about 45 min.

3. *Example word problems*—After collecting the homework, the experimenter distributed booklets containing worked-out examples and practice word problems. The booklets were prepared individually to match each student's pretest condition (e.g., students who had received money problems on their pretest received money problems as examples). Students studied the worked-out examples and the practice problems, checking them with the sets that appeared at the end of each booklet. After 20 min, the example booklets were collected and students were given a posttest matching their content condition. This posttest consisted of the set of four problems matched to the set used as the pretest. Students were allowed 20 min to complete their posttest.

4. *Transfer session*—This session was conducted separately (in two different classrooms) for the "physics" students and for the "money" and "people" students. The physics students were asked to solve some more algebra problems, in order to help the research team develop instructional materials in math. Half of the physics students received the money transfer set and half received the people transfer set. In each case the transfer set consisted of eight problems: both sets of four for the relevant content, with the two sets presented in a random order. The money and people students were asked to take a physics test to help the research team develop instructional materials in physics. This test consisted of the eight motion problems (two sets

of four). Two simple filler problems involving calculation of distance traveled preceded the eight transfer problems. All students were given 40 min to work on the transfer task. Afterwards, the students were debriefed, and the similarities between the algebra and the physics topics were explained.

Results and Discussion

Table 5 summarizes the results with respect to both problem-solving success (percentage correct) and frequency of applying the learned method to the solution of the word problems (scored as in Experiment 1). Because no significant differences were obtained between the money and people groups, data from these two conditions are collapsed in Table 5. Data for the two types of arithmetic-progression problems (sequences and series) are presented separately.

The data were analyzed by an analysis of variance, with problem type (sequences or series), training condition (money/people or motion content), and test (pretest, posttest, or transfer) as factors. The effect of the test was highly significant for both percentage correct, $F(2, 40) = 39.0$, $p < .001$, $MS_e = .23$, and for use of the learned method, $F(2, 40) = 159$, $p < .001$, $MS_e = .29$. The test factor was examined in more detail using orthogonal contrasts (pretest vs. posttest and transfer, and posttest vs. transfer). Collapsing over training conditions, a highly significant increase was obtained in both percentage correct, $t(40) = 17.4$, $p < .001$, and use of the learned method, $t(40) = 35.2$, $p < .001$, from the pretest to posttest and transfer. The posttest and transfer tests did not differ significantly in percentage correct, $t(40) = 1.4$. However, the learned method was used more frequently on the posttest than the transfer test, $t(40) = 5.91$, $p < .01$, indicating that transfer to examples with novel content was incomplete.

Overall percentage correct was higher for sequence than for series problems, $F(1, 20) = 23.1$, $p < .001$, $MS_e = .30$. In addition, a marginally significant interaction was obtained between problem type and test, $F(2, 40) = 2.98$, $p = .06$, $MS_e = .30$, reflecting a trend for the increase in percentage correct from the pretest to the posttest and transfer tests to be significantly larger for the series problems. As noted with regard to Experiment 1, the learned method provides a greater increase in ease of solution for the series problems than for the sequence problems. Use of the learned method did not differ significantly across the two problem types.

The results demonstrate that algebra training, even when instantiated by examples of word problems drawn from a single content domain, results in very high transfer to isomorphic problems used on a novel content domain. Indeed, whereas none of the subjects applied the learned method on the pretest, all of them applied it to at least some transfer problems. Thus, the provision of disparate training examples, as in Experiment 1, is not crucial to obtaining transfer of algebraic procedures. Because Experiment 2 did not include a multiple-content learning condition, we cannot conclude that diversity of examples would not provide some additional benefit. The degree of transfer produced by single-content training, however, is sufficiently high that ceiling effects would make it difficult to observe any added benefit of multiple training contents. Although generalization by intersection of features may play some role in learning from examples in algebra, it is clear that a more knowledge-based mechanism is operative. At least for students reasonably familiar with arithmetic and algebra, the specific content of new types of word problems is not treated as an important condition for applying the arithmetic-progression methods. Meta-level knowledge of the types of information that are relevant to the solution of algebraic word problems apparently provides a belief system that leads learners to avoid including content-specific details in procedures derived from training examples.

Moreover, the amount of learning and transfer did not differ across the money/people and motion training conditions ($F < 1$). Just as much learning and transfer was produced by training on motion problems as by training on the other content domains. Thus, our previous results showing lack of spontaneous transfer from physics to algebra cannot be attributed to either some intrinsic difficulty in learning from motion examples or to use of a single training content per se. Rather, it appears that transfer from physics to other domains is blocked by the embedding of the physics equations within a specific content domain. In physics, unlike algebra, content cues involving motion concepts are encoded as conditions that restrict the applicability of physics equations. Experiment 3 was designed to examine the role of such conceptual embedding in impeding interdomain transfer.

Table 5
Performance Measures for Algebra and Physics Problems Under Single-Content Algebra Training in Experiment 2

	% application of learned method			% correct solutions		
Measure	Sequences %	Series %	Total %	Sequences %	Series %	Total %
Money or people training ($N = 12$)						
Pretest	0	0	0	54	17	35
Posttest	100	96	98	92	83	88
Transfer	71	79	75	75	58	67
Physics training ($N = 10$)						
Pretest	0	0	0	65	20	42
Posttest	100	90	95	85	55	70
Transfer	85	85	85	95	80	88

Note. The percentages are based on the average number of problems per subject, out of two possible sequences and two possible series problems for the pretests and posttests, and out of four sequences and four series problems for the transfer tests.

Experiment 3

The results of Experiments 1 and 2 indicate that instruction in algebraic procedures is readily transferred to novel problems with physics content. Transfer is sufficiently robust that it occurs even when the algebra instruction is accompanied by examples of word problems with only a single content, and the physics transfer problems are presented on a later day. Experiment 3 was performed to investigate the limits of algebra transfer and to test a leading hypothesis regarding the conditions that limit transfer. The results of Experiment 2 indicate that students at the experience levels tested have learned that domain-specific details in algebra word problems are typically irrelevant to the applicability of algebraic equations. To the extent that the physics problems presented on the transfer test are viewed as algebra word problems, the motion concepts may be largely ignored, making transfer of the algebraic procedures relatively easy. This is particularly likely in the experiments reported so far because the physics problems are presented after only minimal introduction to physics as a content topic.

If the above analysis is correct, transfer of algebraic procedures to physics problems should be reduced if the latter are embedded in a context that emphasizes the potential relevance of motion concepts to the problem solutions. Experiment 3 tested this hypothesis by varying the context in which the physics transfer problems were embedded. One group received the problems with only a minimal introduction to physics, as in the previous experiments; a second group received them after a more extensive introduction to the general topic of physics; and a third group received the transfer problems after an introduction to the specific motion concepts pertinent to constant-acceleration problems. If conceptual embeddedness interferes with cross-domain transfer, then at least the third of these groups should exhibit lessened spontaneous transfer from prior instruction in arithmetic progressions.

Method

Subjects. Subjects were 38 9th- and 10th-grade students from private and public high schools in Pittsburgh who had not yet studied either the topic of arithmetic progressions or the topic of motion in a straight line with constant acceleration. All students came from above-average math classes. The subjects included 8 male and 30 female students, who were paid for participating in the study.

Materials. Two sets of algebra problems and one set of physics problems, similar to those used in Experiment 1, were used in Experiment 3. Each set consisted of four problems: The first two had the structure of arithmetic sequences and the last two had the structure of arithmetic series. The two sets of arithmetic-progressions problems served as a pretest and as a posttest for the algebra groups. The role of each set (pretest or posttest) was counterbalanced for subjects within each experimental group. The set of constant-acceleration problems served as a transfer set for the algebra groups and as a pretest for the physics control group.

The instructional materials were based on those used in Experiment 1. In addition, a five-page general introduction to physics, without reference to accelerated-motion concepts, was taken from the first introductory chapter from a high school physics text (Murphy et al., 1982).

Procedure. All subjects were recruited for a study described as directed at the development of new instructional materials for high school mathematics and science curricula. All of the students expected to study both an algebra and a physics chapter. Thirty-two subjects were randomly assigned in approximately equal numbers to three experimental conditions. These students studied the algebra chapter and were subsequently tested on constant-acceleration problems. The three groups differed solely with respect to the context in which the physics transfer problems were subsequently presented. In addition, 6 subjects were assigned to a control group that solved a set of physics problems without any prior training. This group provided a baseline for comparing the performance of the algebra-trained subjects on physics transfer problems.

For the experimental groups, the experiment was conducted in a single two-part session. The initial part of the session lasted about 3 hr and consisted of a pretest on algebraic-progression problems, studying the algebra chapter, and a posttest on the matched set of algebra problems. The experimenter provided feedback only regarding the solutions to the practice problems included in the chapter; no feedback was provided for either the pretest or the posttest problems. Up to this point, the session was identical for the three algebra-trained groups.

After a 15-min refreshment break, all subjects in the experimental groups proceeded to the physics transfer test. The three groups differed in the context in which the transfer problems were embedded. The context in the heading-embedded conditions was identical to that used in the comparable transfer conditions in Experiments 1 and 2. Students in this condition immediately proceeded with the transfer test (four constant-acceleration problems). The physics problems were presented as a pretest to the study of the physics chapter and were introduced by a short paragraph describing the nature of problems dealing with constant acceleration.

In the physics-embedded condition, students started by reading the aforementioned five-page general introduction to physics. This chapter introduced the role of hypotheses, theories, and experimentation in science, including examples of famous theories such as that of Galileo. It did not discuss concepts related to accelerated motion. After reading the chapter, subjects were informed that they were about to study a chapter dealing with accelerated motion and were asked to solve the constant-acceleration problems as a pretest.

Subjects in the motion-embedded condition began by reading a modified version of the physics chapter dealing with accelerated motion that was used in the physics-instructed condition of Experiment 1. They read the first five pages of the chapter, which included all the concept definitions, examples, and explanations (such as the distinction between uniform and accelerated motion). This material ended prior to the section introducing the mathematical analysis of motion; thus, no formulas for accelerated motion were introduced. After reading this first part of the motion chapter, the students were asked to solve a pretest of constant-acceleration problems.

The transfer part of the session lasted about 20 min for the heading-embedded conditions and about 30 min for the physics-embedded and motion-embedded conditions. Students talked out loud during their pretest, posttest, and transfer problem-solving, and their entire problem-solving protocol was tape-recorded for later analysis.

Results and Discussion

As in the previous experiments, the major dependent measure of transfer in Experiment 3 was the frequency with which the learned algebraic procedures were applied to the structurally isomorphic but unfamiliar physics problems. Table 6 presents the percentage of problems solved by the learned

Table 6
Performance Measures for Algebra Training and Physics Transfer Problems for the Different Embedding Conditions in Experiment 3

Condition	% application of learned method			% correct solutions			Solution time per problem (minutes)		
	Sequences	Series	Total	Sequences	Series	Total	Sequences	Series	Total
Motion embedded ($N = 12$)									
Pretest	0	0	0	75	27	51	2.10	2.16	2.13
Posttest	92	100	96	88	83	85	1.80	2.33	2.07
Transfer	33	38	35	88	40	64	1.12	1.96	1.54
Physics embedded ($N = 12$)									
Pretest	0	0	0	71	25	48	2.08	2.93	2.50
Posttest	100	92	96	96	75	86	1.99	2.72	2.36
Transfer	46	71	59	83	65	74	1.23	1.62	1.43
Heading embedded ($N = 8$)									
Pretest	0	0	0	62	34	48	1.65	2.06	1.86
Posttest	100	100	100	94	84	89	1.71	2.72	2.21
Transfer	50	69	59	69	66	67	1.18	1.65	1.41
Control ($N = 6$)									
Pretest	0	0	0	67	29	48	1.35	1.72	1.54

Note. The percentages are based on the average number of problems per subject out of 2 possible sequences and 2 possible series problems.

method and the percent solved correctly for the various conditions, as well as adjusted mean solution times, calculated as in Experiment 1. The arithmetic-progression equations were never used by any subjects prior to algebra training, as would be expected. After training, all of the algebra students applied the learned methods to the posttest algebra problems. All students thus acquired the new method for the solution of word problems dealing with the learned domain.

With respect to transfer, as measured by use of the learned method on the unfamiliar physics problems, the learned algebraic methods were applied to 52% of the physics problems (collapsing over all three context conditions). This figure is of course greater than the zero frequency obtained on the algebra pretest and for the untrained physics control group, replicating the previous findings. The overall differences in use of the learned method across the three tests given to experimental subjects were highly significant, $F(2, 58) = 119$, $p < .001$, $MS_e = .39$. A planned comparison indicated that the learned methods were used significantly less often on the physics transfer test (52%) than on the algebra posttest (98%), $t(29) = 12.3$, $p < .01$, indicating that transfer was incomplete. A significant interaction was obtained between test and problem type, $F(2, 58) = 3.76$, $p < .05$, $MS_e = .14$, which reflects greater transfer of the learned method for the more difficult series problems than for the sequence problems.

Although all three context conditions clearly produced transfer of the learned method, there was some evidence that less transfer was obtained in the motion-embedded condition than in the physics-embedded and heading-embedded conditions. Differences between frequencies of applying the learned method to transfer problems were examined using planned comparisons. Transfer frequency was significantly less for subjects in the motion-embedded condition than for subjects in the physics-embedded and heading-embedded conditions, $t(29) = 5.08$, $p < .01$, $MS_e = .53$; the latter two conditions did not differ from each other, $t(29) < 1$. Only 5 of the 12 subjects in the motion-embedded condition noticed the relevance of the arithmetic progressions to the solution of the constant-acceleration problems, and only 4 subjects actually applied the learned equations to at least one of the physics problems. In contrast, 10 of the 12 subjects in the physics-embedded condition and 6 of the 8 subjects in the heading-embedded condition applied the learned equations to the physics problems. The lesser frequency of subjects showing transfer in the motion-embedded condition versus the other two conditions was significant, $\chi^2(1, N = 32) = 4.88$, $p < .05$.

With respect to the percentage of correct solutions, we first compared the success of the experimental groups on the algebra pretest with the success of the control group on the physics pretest. Because there was no difference in the baseline success for the pretest problems regardless of their content, the subsequent analyses were performed within subjects, comparing success on the physics transfer problems directly to success on the algebra pretest. The effect of test was highly significant, $F(2, 58) = 23.0$, $p < .001$, $MS_e = .38$, as was the interaction between test and problem type, $F(2, 58) = 5.01$, $p < .01$, $MS_e = .27$, indicating a greater increase in percentage correct with training for the more difficult series problems. Neither the main effect nor any interactions involving training condition were significant.

Mean solution times, adjusted in the same manner as in Experiment 1, are also presented in Table 6. Solution times were longer for series problems than for sequence problems, $F(1, 29) = 15.4$, $p < .001$, MS_e (in seconds) = 14,899, and solution times varied significantly across tests, $F(2, 58) = 7.52$, $p < .001$, $MS_e = 21{,}149$. Unlike the results of Experiment 1, solution times were substantially lower for the physics transfer test than for the algebra posttest. However, the solution times for the physics control group were just as short as for the transfer group, suggesting that the time differences reflected intrinsic differences between the algebra and physics test problems, rather than the impact of training. Various procedural differences between Experiment 3 and Experiment 1 (e.g., the training materials for Experiment 3 did not include test problems, and the posttest preceded rather than followed the transfer test) prevent meaningful cross-experiment comparisons.

Overall, the results of Experiment 3 indicate that transfer of algebraic procedures to unfamiliar physics problems is somewhat impaired when the latter are presented in the context of instruction about the relevance of the domain-specific motion concepts. Even though no information about solution procedures for motion problems was provided, emphasis on motion concepts created a context in which such concepts were viewed as likely to be important to problem solutions. Consequently, students were less likely to see that their previously acquired knowledge of arithmetic progressions was applicable to this new content.

We found no evidence, however, that a general discussion of the topic area, as was provided in the physics-embedded condition, is sufficient to impair interdomain transfer. Because students in the physics-embedded condition were not given any specific and directly relevant information regarding constant-acceleration problems, they apparently treated these problems as members of the category of arithmetic-progression problems. Because the length of the intervening physics instruction was equated for the physics-embedded and motion-embedded conditions, sheer amount of material is not the critical factor. Rather than amount of contextual information, or general physics context, what does seem to somewhat restrict transfer from algebra to physics is the introduction of physics information directly relevant to the concepts mentioned in the physics word problems. The additional meaning given to the various variables presented in these problems makes it harder to strip away what now seems to be relevant content and to recognize the structural similarity of the new problems to those for which the algebraic methods had been initially applied.

It should be emphasized that although transfer of algebraic procedures was impaired in the motion-embedded condition, it was by no means eliminated. A substantial number of subjects succeeded in noticing the relevance of the learned methods to the novel content despite the intervening domain-specific instructional material. Also, an analysis of the talk-aloud protocols from the transfer sessions, focusing on the initial recognition of the relevance of the arithmetic-progressions equations to the physics problems, revealed that in all three experimental conditions students applied the algebraic equations in the same straightforward manner as did the algebra students in Experiment 1. (Because these analyses essentially replicated the qualitative findings of the similar analyses performed for Experiment 1, no protocol data are presented for Experiment 3.) Thus, although fewer students in the motion-embedded condition recognized the applicability of the previously learned algebraic equations, those students who did transfer did so not by explicit use of analogy, but rather by treating the physics problems as simple new cases of the familiar arithmetic progression problems. In addition, for students who did not transfer, protocols revealed no evidence of especially thorough analysis of the physical situation, or of exceptional sensitivity to the units of motion. No differences in qualitative analysis of the problem situation were apparent when the protocols of the nontransferring students were compared with those of students who did transfer or when compared with their own solutions to algebra problems dealing with a familiar content. Overall, the negative impact of contextual embedding during transfer appears to be substantially less than the impact of embedding the initial learned procedures in the context of relevant domain-specific concepts, as occurred for physics-trained subjects in Experiment 1.

General Discussion

Summary and Implications

The present experiments provide clear evidence regarding the conditions under which instruction in formal problem-solving procedures yields transfer to word problems with novel content. Training in algebraic equations for solving arithmetic-progression problems, coupled with exposure to example word problems, allowed robust transfer to unfamiliar but isomorphic constant-acceleration problems in physics. The problem-solving protocols of the algebra subjects in Experiments 1 and 3 indicated that the physics problems were viewed as new instances of the learned category of arithmetic-progression problems. Transfer was obtained even when the initial examples used in algebraic instruction were drawn from a single content area, such as money problems (Experiment 2). It appears that students with a moderate level of general knowledge about the typical conditions of applicability for mathematical procedures are able to effectively "screen out" content-specific details in algebra word problems and hence learn relatively abstract applicability conditions for algebraic procedures even when the content of the training examples is overly restricted.

In contrast to the robust transfer observed from algebra to physics, the results obtained with physics students in Experiment 1 indicated that transfer between the two isomorphic domains is strikingly asymmetrical. Students who had learned how to solve constant-acceleration problems in a physics course gave no indication that they recognized any similarity between such problems and arithmetic-progression problems with nonphysics content. The one physics subject in Experiment 1 who noticed the relevance of the constant-acceleration equations to the solution of the arithmetic-progression solution did so by means of an analogical remind-and-map process, rather than by direct recognition that the transfer problems were of a familiar type. Her protocols also provided evidence of a transition from transfer by analogical reminding to transfer by application of generalized rules induced from examples. The striking qualitative differences between her protocol and those of algebra-trained subjects solving physics problems suggest that explicitly analogical transfer between domains is a distinctive and relatively difficult transfer mechanism (Carbonell, 1983, 1986; Gentner, 1983, in press; Gick & Holyoak, 1983; Holyoak & Thagard, in press).

The general lack of transfer from physics to algebra cannot be attributed to any intrinsic property of the physics problems, because algebra training coupled with problems dealing with physics content produced as much transfer as training with other contents of problems (Experiment 2). Rather, in studying physics, students learn that the physical concepts involved in word problems are critical to the applicability of the relevant equations. Accordingly, they do not expect, and fail to recognize, any direct relation between physics problem-solving procedures and isomorphic problems drawn from nonphysics domains. Students' perceptions of the importance of domain concepts in determining transfer was further highlighted by the results of Experiment 3. When students were asked to solve physics word problems in the context of a detailed introduction to motion concepts, the likelihood that they would apply the algebraic procedures they had learned earlier was somewhat reduced.

Directions for Future Research

The results of the present study emphasize the importance of domain-specific expectations in establishing applicability conditions for problem-solving procedures, both during initial encoding and during future access. These applicability conditions in turn mediate the extent of subsequent transfer. Also, since the source domain is usually the better practiced one, the effect of content specificity may be stronger during encoding than during access.

The present results have implications for deciding whether one should teach general and content-free procedures or strategies or teach procedures that are conditionalized on content-specific cues and configurations. Although expertise is generally based on content-specific knowledge, such knowledge is likely to be bound to the domain of expertise. Because there are numerous content-centered domains, such as physics, future research efforts should focus on possible instructional measures to teach these domains in a way that might allow more flexible transfer. To follow Simon's (1980) recommendation: "To secure substantial transfer of skills acquired in the environment of one task, learners need to be made explicitly aware of these skills, *abstracted from their specific task content*" (p. 82, italics added). For example, transfer from physics might be obtained if the physics equations were taught in relation to analogous examples with different contents (e.g., heat problems involving constant increase in temperature).

Content-free domains are more of an exception. The present study provides evidence for extensive transfer from the domain of arithmetic progressions. We do not assume, however, that algebraic procedures are always learned in a content-free fashion. Mathematical procedures are often tailored to a certain subcategory of otherwise isomorphic problems. For example, in typical work problems, the total work is treated as a whole (i.e., 1 unit) and each worker's share is considered to be a fraction of that whole, whereas in typical travel problems the total distance is presented as a quantity (i.e., 90 km). Such focus on the specifics of the solution procedure, when confounded with a single content for example problems, may indeed obscure higher level structural similarity. Studies on categorization of algebra word problems (e.g., Hinsley et al., 1977; Mayer, 1981) and studies dealing with the use of example solutions (Reed, 1987) indicate that algebra students are very sensitive to various content cues. There is also evidence, however, that good students (Krutetskii, 1976; Silver, 1981) or students who received appropriate training (Schoenfeld & Herrmann, 1982) sort word problems more according to their structure and solution method than according to their content. It seems, then, that under appropriate training conditions, solution procedures for algebra word problems can often be accessed without dependence on content cues. In the present study, students received rather extensive training in the general structure of arithmetic progressions, yielding representations that were relatively content free.

The results of Experiment 3, however, begin to delimit the boundary conditions on spontaneous transfer of algebraic knowledge. There is evidence that procedures learned as methods of solving word problems are not readily applied to more realistic problems in which simple keywords do not suffice to cue relevant equations (di Sessa, 1982). It is possible that a content-free encoding of algebraic procedures needs to be further supplemented by training in abstracting new target problems from their rich contextual constraints. More work remains to be done to explore this possibility.

Another important question that remains to be investigated is whether the effect of context specificity during the initial encoding of the problem-solving procedures is limited to preventing spontaneous access, or whether it also prevents the application of these procedures even when people are informed about their relevance. An empirical study of this question would have to include an exploration of the effects of active interventions at the time of the transfer tests. In the present study, subjects were given no direct cues that the initially learned procedures were applicable to the novel transfer problems; indeed, the latter were presented in the context of instruction in a different topic. We do not know whether the content specificity that prevents spontaneous transfer from physics to isomorphic problems with nonphysics content would still preclude transfer even if the relevance of the physics training was highlighted rather than obscured. It would be useful to examine transfer under conditions in which subjects are actively encouraged to apply the learned physics methods to transfer problems and perhaps are given guidance by a teacher in performing the required mapping between transfer problems and the learned equations.

Whatever the outcome of such further investigations, our present findings should temper the prevalent opinion among cognitive scientists that spontaneous transfer between relatively dissimilar problems is invariably difficult to obtain. Many previous studies have sought transfer, often unsuccessfully, under the most difficult possible conditions, with only a single isolated example as the basis for potential transfer (see Brown et al., 1986). The present study indicates that, given learning conditions conducive to acquisition of relatively general applicability conditions for problem-solving procedures, people are able to readily recognize structurally similar examples despite their unfamiliar surface features. We have begun, but hardly completed, the larger task of exploring the boundary conditions on transfer in the presence of various forms of content-specific interference.

References

Anderson, J. R. (1983). *The architecture of cognition.* Cambridge, MA: Harvard University Press.

Bassok, M., & Holyoak, K. J. (1985, November). *Transfer between isomorphic topics in algebra and physics.* Paper presented at the annual meeting of the Psychonomic Society, Boston.

Bransford, J., Sherwood, R., Vye, N., & Rieser, J. (1986). Teaching thinking and problem solving. *American Psychologist, 41,* 1078–1089.

Brown, A. L., Kane, M. J., & Echols, K. (1986). Young children's mental models determine transfer across problems with a common goal structure. *Cognitive Development, 1,* 103–122.

Caramazza, A., McCloskey, M., & Green, B. (1981). Naive beliefs in "sophisticated" subjects: Misconceptions and trajectories of objects. *Cognition, 9,* 117–123.

Carbonell, J. G. (1983). Learning by analogy: Formulating and generalizing plans from past experience. In R. Michalski, J. G. Carbonell, & T. M. Mitchell (Eds.), *Machine learning: An artificial intelligence approach.* Palo Alto, CA: Tioga Press.

Carbonell, J. G. (1986) Derivational analogy: A theory of reconstructive problem solving and expertise acquisition. In R. Michalski, J. G. Carbonell, & T. M. Mitchell (Eds.), *Machine learning: An artificial intelligence approach* (Vol. 2, pp. 371–392). Los Altos, CA: Kaufmann.

Cheng, P. W., Holyoak, K. J., Nisbett, R. E., & Oliver. L. M. (1986). Pragmatic versus syntactic approaches to training deductive reasoning. *Cognitive Psychology, 18,* 293–328.

Chi, M. T. H., Feltovich, P. J., & Glaser, R. (1981). Categorization and the representation of physics problems by novices and experts. *Cognitive Science, 5,* 121–152.

di Sessa, A. A. (1982). Unlearning Aristotelian physics: A study of knowledge-based learning. *Cognitive Science, 6,* 37–75.

Dolciani, M. P., Wooton, W., Beckenbach, E. F., & Sharon, S. (1983). *Algebra 2 and trigonometry.* Boston: Houghton Mifflin.

Fong, G. T., Krantz, D. H., & Nisbett, R. E. (1986). The effects of statistical training on thinking about everyday problems. *Cognitive Psychology, 18,* 253–292.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7,* 155–170.

Gentner, D. (in press). The mechanisms of analogical reasoning. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning.* Cambridge, MA: Cambridge University Press.

Gentner, D., & Gentner, D. R. (1983). Flowing waters or teeming crowds: Mental models of electricity. In D. Gentner & A. L. Stevens (Eds.), *Mental models* (pp. 99–129). Hillsdale, NJ: Erlbaum.

Gick, M. L., & Holyoak, K. J. (1980). Analogical problem solving. *Cognitive Psychology, 12,* 306–355.

Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology, 15,* 1–38.

Hayes, J. R., & Simon, H. A. (1977). Psychological differences among problem isomorphs. In N. J. Castellan, Jr., D. B. Pisoni, & G. R. Potts (Eds.), *Cognitive theory* (Vol. 2, pp. 21–41). Hillsdale, NJ: Erlbaum.

Hayes-Roth, F., & McDermott, J. (1978). An interference matching technique for inducing abstractions. *Communications of the ACM, 21,* 400–411.

Hinsley, D. A., Hayes, J. R., & Simon, H. A. (1977). From words to equations: Meaning and representation in algebra word problems. In M. A. Just & P. A. Carpenter (Eds.), *Cognitive processes in comprehension* (pp. 89–106). Hillsdale, NJ: Erlbaum.

Holyoak, K. J. (1985). The pragmatics of analogical transfer. In G. H. Bower (Ed.), *The psychology of learning and motivation* (Vol. 19, pp. 59–87). New York: Academic Press.

Holyoak, K. J., & Koh, K. (1987). Surface and structural similarity in analogical transfer. *Memory & Cognition, 15,* 332–340.

Holyoak, K. J., & Thagard, P. (in press). A computational model of analogical problem solving. In S. Vosniadou & A. Ortony (Eds.), *Similarity and analogical reasoning,* Cambridge, MA: Cambridge University Press.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review, 92,* 109–129.

Krutetskii, V. A. (1976). Characteristics of information retention (mathematical material) by mathematically capable pupils. In J. Kilpatrick & I. Wirshup (Eds.), *The psychology of mathematical abilities in school children* (pp. 295–301). Chicago: University of Chicago Press.

Larkin, J. H., McDermott, J., Simon, D., & Simon, H. A. (1980). Expert and novice performance in solving physics problems. *Science, 208,* 1335–1342.

Mayer, R. E. (1981). Frequency norms and structural analysis of algebra story problems into families, categories and templates. *Instructional Science, 10,* 135–175.

McCloskey, M. (1983). Intuitive physics. *Scientific American, 248,* 122–130.

Murphy, J. T., Hollon, J. M., & Zitzewitz, P. W. (1982). *Physics: Principles and problems.* Columbus, OH: Bell & Howell.

Reed, S. K. (1987). A structure-mapping model for work problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 13,* 124–139.

Reed, S. K., Dempster, A., & Ettinger, M. (1985). Usefulness of analogous solutions for solving algebra word problems. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 11,* 106–125.

Reed, S. K., Ernst, G. W., & Banerji, R. (1974). The role of analogy in transfer between similar problem states. *Cognitive Psychology, 6,* 436–450.

Ross, B. H. (1984). Remindings and their effects in learning a cognitive skill. *Cognitive Psychology, 16,* 371–416.

Ross, B. H. (1987). This is like that: The use of earlier problems and the separation of similarity effects. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 13,* 629–639.

Schoenfeld, A. (1985). *Mathematical problem solving.* New York: Academic Press.

Schoenfeld, A. H., & Herrmann, D. J. (1982). Problem perception and knowledge structure in expert and novice mathematical problem solvers. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 5,* 484–494.

Silver, E. A. (1981). Recall of mathematical problem information: Solving related problems. *Journal of Research in Mathematics Education, 12,* 54–64.

Simon, H. A. (1980). Problem solving and education. In D. T. Tuma & R. Reif (Eds.), *Problem solving and education: Issues in teaching and research* (pp. 81–96). Hillsdale, NJ: Erlbaum.

Spencer, R. M., & Weisberg, R. W. (1986). Is analogy sufficient to facilitate transfer during problem solving? *Memory & Cognition, 14,* 442–449.

Winston, P. H. (1975). Learning structural descriptions from examples. In P. H. Winston (Ed.), *The psychology of computer vision* (pp. 157–209). New York: McGraw-Hill.

Whitehead, A. N. (1929). *The aims of education.* New York: Macmillan.

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