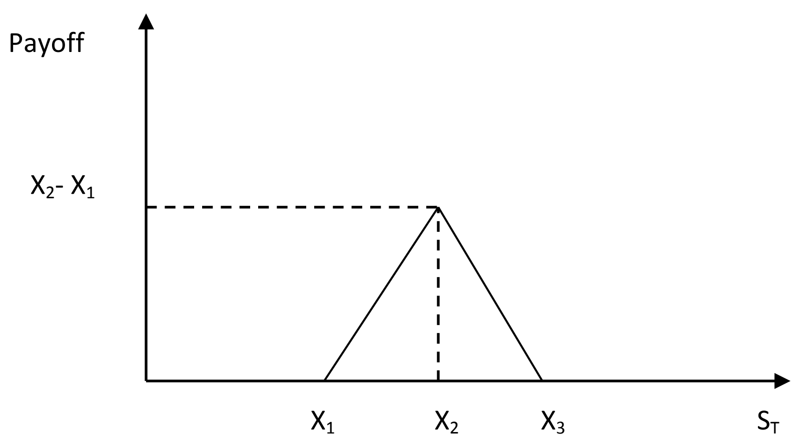
Olympiad Solutions for problems

**Problem 1**

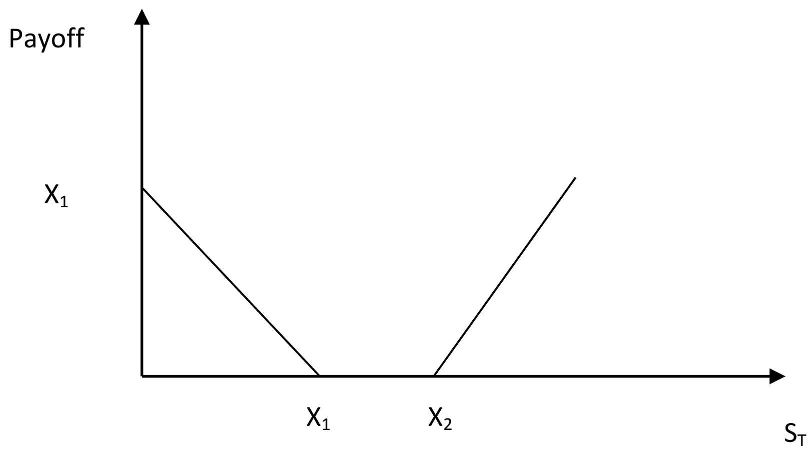
**a) Butterfly spread**

| Position | S<X1 | X1<S<X2 | X2<S<X3 | X3<S |
|---|---|---|---|---|
| Long call X1 | 0 | S-X1 | S-X1 | S-X1 |
| Short 2 calls X2 | 0 | 0 | -2(S-X2) | -2(S-X2) |
| Long call X3 | 0 | 0 | 0 | S-X3 |
| Total | 0 | S-X1 | 2X2-X1-S | (X2-X1)-(X3-X2)=0 |



**b) Vertical combination**

| Position | S<X1 | X1<S<X2 | S>X2 |
|---|---|---|---|
| Long call X2 | 0 | 0 | S-X2 |
| Long put X1 | X1-S | 0 | 0 |
| Total | X1-S | 0 | S-X2 |



Problem 2

Solution

Assume all vars in deviation from

a) Model: $\log w = \cancel{\beta_0} + \beta_1 \, educ + \beta_2 \, abil + \varepsilon$

If abil is omitted:

$$\hat{\beta}_1 = \beta_1 + \frac{\sum educ(\beta_2 \, abil + \varepsilon)}{\sum educ^2}$$

$$= \beta_1 + \beta_2 \frac{\sum educ \, abil}{\sum educ^2} + \frac{\sum educ \, \varepsilon}{\sum educ^2}$$

↳ plim = 0

$\hat{\gamma}_1 \rightarrow$ slope coeff in:

$abil = \gamma_1 \, educ + v$

So, $\boxed{\text{plim } \hat{\beta}_1 = \beta_1 + \beta_2 \gamma_1}$

↳ Bias proportional to the effect of abil on earnings and educ. on abil.

b) $educ = \underset{\text{true value}}{educ^*} + e \longrightarrow$ measurement error.

$E(e) = 0$

$Var(e) = \sigma_e^2$

$Cov(educ^*, e) = 0$

$Cov(e, \varepsilon) = 0$ see point (a)

$$\log w = \beta_1 \, educ + \varepsilon - \beta_1 e$$

$$\hat{\beta}_1 = \beta_1 + \frac{\sum educ(\varepsilon - \beta_1 e)}{\sum educ^2}$$

$$\text{plim } \hat{\beta}_1 = \beta_1 \left(1 - \frac{Cov(educ, e)}{VAR(educ)}\right) = \beta_1 \left(\frac{\sigma^2_{educ^*} + \cancel{\sigma_e^2} - \cancel{\sigma_e^2}}{\sigma^2_{educ^*} + \sigma_e^2}\right)$$

SIGNAL-NOISE RATIO

c) $\beta_1 = 0.05$

$\beta_2 = 0.02$

From (a): $\hat{\beta}_1 = 0.05 + 0.02 \cdot \frac{1.8}{9} = 0.054$

Thus, using (b), we know that the approximate SIGNAL-NOISE must be of order $\frac{0.05}{0.054} \approx 0.926$, which is a reasonable amount of noise.

**Problem 3**

Solution:
Expand the lhs as first-order Taylor series around $x$ to get

$$u(x-\rho) \simeq u(x) + u'(x)(x-\rho)\frac{1}{1!} = u(x) - u'(x)\rho$$

For all values of $z_s$, expand the rhs as second-order Taylor series to get

$$u(x+z_s) \simeq u(x) + u'(x)z_s\frac{1}{1!} + u''(x)z_s^2\frac{1}{2!}$$

Sum them over $s$ and equate the two approximations to obtain

$$\begin{aligned} u(x) - u'(x)\rho &= u(x) + \sum_s p_s u'(x) z_s + \sum_s p_s u''(x) z_s^2 \frac{1}{2} \Rightarrow \\ -u'(x)\rho &= 0 + \frac{u''(x)}{2}\sigma^2 \Rightarrow \rho = -\frac{u''(x)}{u'(x)}\frac{\sigma^2}{2} = \frac{1}{2}A\sigma^2 \end{aligned}$$

because $\sum_s p_s z_s = \bar{z} = 0$ and $\sum_s p_s z_s^2 = \sum_s p_s (\bar{z} - z_s)^2$. It follows that risk premium is related to the (negative of) coefficient of absolute risk aversion $A = -\frac{u''(x)}{u'(x)}$, which is the Arrow-Pratt measure. Exact conditions are that the third absolute central moment of $z$ is of order smaller than $\sigma_z^2$, denoted $o(\sigma_z^2)$, and that second absolute central moment of $\rho$ is of order at most $O(\rho^2) < o(\sigma_z^2)$.

**Problem 4**

a) The value marginal product of labor in both industries must be equal to wage: $p_G MPL_G = w = p_R MPL_R$. Using the standard formulas for the MPL, we obtain $0.5\times 8^{0.5} L_G^{-0.5} = w = 8^{0.5} L_R^{-0.5}$. By combining this with $L_G + L_R = 40$, we obtain $L_G=8$, $L_R=32$, $w=0.5$. The outputs are $q_G = K^{0.5} L_G^{0.5} = 8$, $q_R = T^{0.5} L_R^{0.5} = 16$. The returns to other factors are $r = 0.5\times K^{-0.5} L_G^{0.5} = 0.5$ for capital and $s = 0.5\times T^{-0.5} L_R^{0.5} = 2$ for land.

b) The national GDP is (production method) $p_G q_G + p_R q_R = 40$. Given Leontief preferences, consumers buy guns and roses in bundles; the cost of each bundle is $p_G + p_R = 3$. Therefore, the total number of bundles (also total consumption of each good) is 40/3. Since 40/3>8 (supply of guns), the guns are imported with the import quantity 40/3-8=16/3. The worker aggregate welfare is total labor income divided by the bundle cost, $W_L = w\times L/(p_G+p_R) = 20/3$. Likewise, $W_K = r\times K/(p_G+p_R) = 4/3$ and $W_T = s\times T/(p_G+p_R) = 16/3$.

c) Labor market clearance: $0.5(1+t)\times 8^{0.5} L_G^{-0.5} = w = 8^{0.5} L_R^{-0.5}$; $L_G + L_R = 40$. Solution: $L_G = 40(1+t)^2/(4+(1+t)^2)$; $L_R = 160/(4+(1+t)^2)$; $q_G = 8(1+t)5^{0.5}(4+(1+t)^2)^{-0.5}$; $q_R = 16\times 5^{0.5}(4+(1+t)^2)^{-0.5}$. Factor prices: $w = 20^{-0.5}(4+(1+t)^2)^{0.5}$; $r = (1+t)^2 5^{0.5}/2(4+(1+t)^2)^{-0.5}$; $s = 2\times 5^{0.5}(4+(1+t)^2)^{-0.5}$. The total welfare of all groups is equal to GDP in domestic prices divided by the cost of the bundle, $W = W_L + W_K + W_T = ((1+t)q_G + 2q_R)/(1+t+2) = 8\times 5^{0.5}(4+(1+t)^2)^{0.5}/(3+t)$. This welfare is divided between groups proportionately to their income: $W_L = w\times L/(w\times L + r\times K + s\times T)\times W = 4\times 5^{0.5}(4+(1+t)^2)^{0.5}/(3+t)$.

Likewise, $W_K$=r×K/(w×L+r×K+s×T)×W=$4\times5^{0.5}(1+t)^2(4+(1+t)^2)^{-0.5}/(3+t)$;
$W_T$=s×T/(w×L+r×K+s×T)×W=$16\times5^{0.5}(4+(1+t)^2)^{-0.5}/(3+t)$.

d) The quantity of guns demanded is numerically equal to W, while the supply is given by $q_G$. The prohibitive tariff is such that the two are equal to each other, which is true when t=1.

e) By differentiating (logs of) $W_L$, $W_K$, $W_T$, W with respect to t, one can show that $W_K$ is rising on [0,1] while others are decreasing. Therefore, the optimum tariff is t=0.

**Problem 5**

a) Ann has to do Bayesian updating of her prior on having picked 2 aces (1 of 6 combinations, so 1/6) after Bob's signal "you picked the ace of y" (y=hearts,diamonds). Without loss of generality, let's focus on the signal "you picked the ace of hearts". Since Bob cannot lie, this signal can come from 3 combinations: ace of hearts with any of the other 3 cards. Each of the 2 combinations with a jack generates the signal with probability 1. The combination with the other ace generates the signal with probability 1/2. Therefore, by Bayes rule, the probability that the signal comes from the last combination is the probability of its realization (1/6) times the consequent probability of generating the signal (1/2), divided by the total probability of receiving the signal (the sum of probabilities of receiving the signal from the three combinations (1/6+1/6+1/12). The result is 1/5, so Ann is willing to bet only if x (the amount of euros put by Bob in the pot) is at least 5.

b) As a monopolist, the optimal price for the first firm is 9 (MC=8=10-2D=MR, D=1, P=9). Since at this price the second firm would make negative profits by producing, it produces zero and the first firm actually behaves as a monopolist.