



Olympiad Examination 2016

ID number: _____

Examination Guidelines

- The exam consists of 5 questions. Answer all questions. Time 180 minutes.
- The exam is graded on a 100-point scale. The marks awarded for each problem are in brackets.
- Write your answers to the booklet provided to you by the examiners.
- You can solve the problems in any order but you must label each problem and its sub-questions clearly and sufficiently. Use a separate page for each problem. You are not allowed to detach sheets from the booklet.
- Answer all questions in English. Graders will ignore any Russian text.
- You may use the last page of your booklet as scrap paper.
- Crossed out writing will not be considered by the grader.
- You are kindly requested to use legible hand writing. The grader will ignore any illegible parts of your paper.

Examination Rules

- You are required to follow all instructions given by the examiners.
- Talking is not allowed under any circumstances.
- During the exam you are allowed to have on your desk two pens (black or blue) and a drink. You are not allowed to bring any written or printed materials into the examination room. Mobile phones and other electronic devices are strictly prohibited in the examination room.
- Detection of any electronic device on you (apart from a watch) will constitute cheating.
- The proctors of the exam are not authorized to answer any questions.
- Exam participants are not allowed to leave the examination room until ready to turn in their work.

I have read and understood the examination rules. I will not cheat, copy or use unauthorized materials or devices.

Signed: _____

Question 1 [20 pts]

You are deciding between two mutually exclusive investment opportunities. Both require the same initial investment of \$10 million. Investment A will generate \$2 million per year (starting at the end of the first year) in perpetuity. Investment B will generate \$1.5 million at the end of the first year and its revenues will grow at 2% per year for every year after that. Assume further that the expected returns of both investments have a sensitivity of 0.8 with respect to the market return. You know about another firm (whose beta is 1.4) that its expected return is 10%. The risk-free rate is 3%.

- Which investment has the higher IRR?
- Use the CAPM model to calculate the expected return (=cost of capital) of the investment projects under consideration.
- Which investment has the higher NPV?
- State the NPV and IRR rules of investment. What would be your decision according to each rule? Why do they imply different decisions? Answer the last question with the help of a graph that shows the relation between NPV and IRR for each of the two projects.
- Use the incremental IRR rule to correctly choose between the investments. In particular, derive the investment rule for this example, and decide in which project to invest using the cost of capital obtained in (b).

Question 2 [20pts]

Suppose the world market for airliners is served by two companies, Airbus and Boeing. The global demand for Airbus aircraft is $q_A = s - b(p_A - p_B)$ where p_A is the price of an Airbus aircraft, p_B is the price of Boeing aircraft. The demand for Boeing is symmetric: $q_B = s - b(p_B - p_A)$. The marginal cost is c , the same for both companies.

- Write down the first-order condition of maximal profit for each company. Find the best response of Airbus to the price of Boeing. Find equilibrium prices, quantities, and profits π_A for Airbus and π_B for Boeing.

Suppose the European government introduces an export tax on Airbus aircraft of t_A per unit. Likewise, the U.S. government introduces an export tax of t_B per unit of Boeing aircraft. Thus, the demand becomes $q_A = s - b(p_A - p_B + t_A - t_B)$ for Airbus and $q_B = s - b(p_B - p_A + t_B - t_A)$ for Boeing.

- Write down the first-order condition of maximal profit for each company. Find the equilibrium prices, quantities, profits for Airbus and Boeing, government tax revenues R_A for Europe and R_B for the U.S., all as functions of tax levels t_A and t_B .

Suppose the European government chooses t_A to maximize Airbus profit plus its own tax revenue, $\pi_A + R_A$. Likewise, the U.S. government chooses t_B to maximize $\pi_B + R_B$. The governments realize that their taxes will affect the prices of both producers, but take each other's tax rate as given.

- Find the best response of the European government to the U.S. tax rate. Find the equilibrium tax rates by both governments. Find the profits of Airbus and Boeing under these tax rates.
- Is $\pi_A + R_A$ higher under the equilibrium tax rate than under the case of no taxation by both governments? Why or why not?

Question 3 [20pts]

Suppose the probability that a person is a smoker is given by the logit model, namely $\Pr(y = 1|x) = \exp(a + \beta x) / [1 + \exp(a + \beta x)]$, where y is a dummy variable taking the value one for smokers and zero for nonsmokers, x is a dummy variable taking the value one for males and zero for females, and α and β are the parameters to be estimated by maximum likelihood (ML). We have 100 observations, of which 10 are smoking males, 15 are smoking females, 35 are nonsmoking males, and 40 are nonsmoking females.

- a) What are the ML estimates of α and β ?
- b) What are the ML estimates of the probability that a male is a smoker and the probability that a female is a smoker?
- c) What is the ML estimate of the probability that a person is a smoker, under the null $H_0: \beta = 0$?

Question 4 [20pts]

Till now the international market of bananas was ruled by the Organization of Bananas Exporting Countries (OBEC), which consists of two countries. Recently, global warming led to the appearance of a third producing country of bananas, which did not join OBEC. The appearance of the new producer worries OBEC that price of bananas will fall abruptly. The international demand for bananas is given by $P = a - bQ$, where Q is the total production in tons. The production cost of one ton of bananas is the same for all countries and equals c .

- a) Find the quantity of bananas produced by each country, the market price and the profits after the appearance of the new producer, assuming that the two OBEC countries behave as a cartel and the third acts alone. Also assume that OBEC preannounces its decisions on production and members are known to commit to those decisions.
- b) A critique on OBEC's policy states that it was good idea for OBEC to act cooperatively when there were no other producers, but with the appearance of the new player it is better for the two OBEC countries to act on their own without prior commitment. Evaluate the statement and find the quantity produced by each country, profits and market price if all three countries act independently.
- c) Assume that in a general situation n countries produce bananas: the newcomer and $n - 1$ belonging to OBEC. Which market structure out of two described previously would be preferred by the newcomer?

Question 5 [20pts]

Consider an economy that lasts for 2 periods, $t = 0, 1$. There is a representative household who is endowed with one unit of time each period. She allocates a proportion of her time to working, and each unit of time spent working produces one unit of consumption good. The consumption good is perishable. She prefers more consumption, but dislikes working as it reduces the time she can allocate to leisure activities. Let $l(t)$ denote the amount of time spent working each period, $w(t)$ be the wage rate and $c(t)$ denote consumption. Assume markets are competitive and prices are flexible. Finally, assume that the household has the following preferences over consumption today and tomorrow:

$$U(c(0), c(1)) = \log(c(0)) + \log(1 - l(0)) + \beta \log(c(1)) + \beta \log(1 - l(1))$$

where $\beta = 1$.

- a) What is the wage rate?
- b) How much will the household consume today and tomorrow?
- c) Suppose there is a government that needs to raise an amount of resources $g(0)$ in the first period. This can be achieved by either taxing labor income in the first period, or by borrowing from the household in the first period and repaying the debt by taxing labor income in the second period. If the government imposes a tax rate of $\tau(t)$, then the household receives $w(t)l(t) - \tau(t)w(t)l(t)$ in labor income. If the government borrows $b(0)$ in the first period, it must repay $b(0)(1 + r(0))$ in the second period. The government has no further incentives or requirements and otherwise wants to maximize the utility of the households. Is it preferable that the government taxes in the first period or borrows in the first period and taxes in the second period to repay the debt, or a combination of the two policies? why?

End of questions - Good luck!