



## ICEF Olympiad Examination 2016

Код: 130

Время выполнения — 180 минут

### Examination Guidelines

- The exam consists of 5 questions. Answer all questions. Time 180 minutes.
- The exam is graded on a 100-point scale. Numbers in brackets indicate the points awarded for each problem.
- Write your answers in the booklet provided to you by the examiners.
- You can solve the problems in any order but you must label each problem and its sub-questions clearly and sufficiently. Use a separate page for each problem. You are not allowed to detach sheets from the booklet.
- Answer all questions in English. Graders will ignore any Russian text.
- You may use the last page of your booklet as scrap paper.
- Crossed out writing will not be considered by the grader.
- You are kindly requested to use legible hand writing. The grader will ignore any illegible parts of your paper.

### Examination Rules

- You are required to follow all instructions given by the examiners.
- Talking is not allowed under any circumstances.
- During the exam you are allowed to have on your desk two pens (black or blue) and a drink. You are not allowed to bring any written or printed materials into the examination room. Mobile phones and other electronic devices are strictly prohibited in the examination room.
- Detection of any electronic device on you will constitute cheating even if the device is off.
- The proctors of the exam are not authorized to answer any questions.
- Exam participants are not allowed to leave the examination room until ready to turn in their work.

*I have read and understood the examination rules. I will not cheat, copy or use unauthorized materials or devices.*

Signed: \_\_\_\_\_

### Question 1 [20p]

Consider the following problem for an airline. There are 100 customers, who can be of two types: high income ( $\theta^H$ , 10 customers) and low income ( $\theta^L$ , 90 customers). The payoff of a customer of type

$\theta = \theta^H, \theta^L$  is  $x + y - \frac{(p(y))^2}{\theta}$ , where:

- $x$  takes value 1 if the customer flies and 0 otherwise;
- $y$  takes value 1 if the customer enjoys business class services and 0 otherwise;
- $p(y)$  is the price of the ticket ( $p(0)$  is the price of an economy class ticket and  $p(1)$  is the price of a business class ticket).

For the airline, an economy class passenger has 0 marginal cost, a business class passenger has a marginal cost of 1/2. Suppose that  $\theta^H = 2$  and  $\theta^L = 1$ . The airline cannot recognize and discriminate the passengers, but it can offer only economy class tickets, only business class tickets, or both.

- a) Prove that the airline prefers to issue ticket(s) so that both types of customers will purchase. [5p]
- b) Argue that for the airline it is optimal to serve low-income customers with an economy class ticket and the high-income customers with a business class ticket. Then find the optimal prices of the two tickets. [10p]
- c) Suppose that low-income customers incur the risk of missing the flight with probability 10%. To still convince them to buy the ticket, what does the airline prefer: a discount on the price or to offer reimbursement in case of no show? Explain. [5p]

### Question 2 [20p]

There are two agents in an economy. They live for one period and get utility from a consumption good  $c$  and real money balances  $\frac{M}{P}$ , where  $M$  is nominal money holdings and  $P$  is the price level, i.e. the price of the consumption good. Assume the utility function of agent  $i = 1, 2$  is  $U_i = \log(c_i) + \log\left(\frac{M_i}{P}\right)$  and agent  $i = 1, 2$  is endowed with  $e_i$  units of the consumption good and  $X_i$  units of nominal money. The agents can trade the consumption good and money if they wish.

- a) What is the demand for consumption good and the demand for nominal money for each agent? [4p]
- b) What are the market clearing conditions? [2p]
- c) What is the equilibrium price level? Do you need to consider the equilibria on both markets to find it? Why? [4p]
- d) Will the agents be better off if their endowments of nominal money double? [2p]

Assume now that there is also a government in this economy. The government does not produce the consumption good but can print any amount of nominal money.

- e) How much money will the government need to print to buy  $g$  units of the consumption good from the market? [4p]
- f) What is the maximum  $g$  that the government can buy with printing money? [4p]

**Question 3 [20p]**

- a) Imagine that there are two countries (A and B) in the world. Each country can produce two goods – food (F) and energy (E). There is only one factor of production – labour – which is not mobile between countries. There are 100 units of labour in country A and 300 units of labour in country B. The production of one unit of food requires one labour unit in county A and two units of labour in country B while the production of one unit of energy requires three units of labour in country A and four units of labour in country B.
- Draw the production possibilities frontiers for both countries. Indicate the intercepts and slope for each line. [2p]
  - Define countries' absolute and comparative advantages in producing food and energy? [3p]
  - If countries can trade freely in both goods, show graphically that this may make both countries better off. [4p]
  - Is it possible that country A is better off but country B is same off due to international trade? If yes, illustrate this on a diagram and show the production and consumption decisions consistent with such situation. If no, prove it. [4p]
- b) What is the basic relation between nominal interest rates in different currencies? [3p]
- c) What is the basic relation between prices of individual goods in different countries? What is the relation between price levels in different countries? [4p]

**Question 4 [20p]**

Researcher John Smith estimated two models for Fertility using a data set on socio-economic indicators of Switzerland at about 1888 (47 observations):

	Model 1	Model 2
(Intercept)	60.304 (4.251)	86.225 (4.735)
Agriculture	0.194 (0.077)	-0.203 (0.071)
Catholic		0.145 (0.030)
Education		-1.072 (0.156)
R-square	0.1	0.6
RSS	6283.1	2567.9

Fertility: standardized female fertility measure; Agriculture: percentage of males involved in agriculture; Education: percentage of educated beyond primary school for draftees; Catholic: percentage of catholic people; (standard errors are in parenthesis).

Estimate of covariance matrix for Model 1:

	(Intercept)	Agriculture
<b>(Intercept)</b>	18.07	-0.2981
<b>Agriculture</b>	-0.2981	0.005885

- Find a 95% confidence interval for coefficient before Agriculture in Model 2. [4p]
- Using appropriate test compare the two models at 5% significance level. Clearly state the null and alternative hypothesis. [6p]
- Find 95% predictive interval for fertility in Model 1 for a region with 50% of males involved in agriculture as occupation. [10p]

### Question 5 [20p]

You want to value the stock of the ABC Corporation on December 31, 2015. ABC just generated annual after-tax earnings per share of \$10.00. Without any additional investment (in excess of recovering depreciated assets), the company would be able to generate these earnings at the end of every year forever. ABC always followed no growth strategy. Tomorrow morning shareholders expect to receive their yearly dividend for the last 2015 year. Size of the dividends equals free cash flows to equity-holders (FCFE). ABC Corporation used to exploit only one business of type A. However, ABC's managers decided to diversify. Today, on December 31, 2015, which is the end the last financial year, managers decided to invest 40% of company net profits generated in 2015 into a different type B business. There will be no such investments in the future. All earnings from investment in type B business are planned to be paid out as dividends. You know that investments in type B assets will bring you an expected 12% per annum. However, required return should be determined using comparable companies.

ABC corporation is an all equity firm. Yesterday your estimates were: current beta of equity was 1.2, the risk-free rate was 4%, and that the expected return on the market portfolio was 14%. Assume the only market imperfection is corporate income tax which is 20%.

Available information on type B business includes the following: You observe a diversified public company XYZ that exploits two types of businesses. One is type B which is exactly like that ABC plans to exploit. The other type C and it comprises 2/3 of XYZ's operating assets. Type C business has beta of 1. Company XYZ is financed with debt and equity. Value of its perpetual riskless debt is \$1000, value of equity is also \$1000. Assume interest tax savings are of the same level of risk as XYZ's debt. You believe CAPM holds. XYZ's equity beta is 1.44.

- a) Determine the value of stock ABC's stock before any changes in operating activities. [5p]
- b) Determine required return on assets of type B business using comparables data. [5p]
- c) Will stock price increase or decrease as a result of offered investment decision? Explain. (Note: do not use your answer in (d) when answering (c)) [4p]
- d) Determine the value of ABC's stock just after the investment decision is made. [6p]

*End of questions - Good luck!*

*(Pages 5 and 6 follow with statistical tables)*

Table entry for p and C is the critical value  $t^*$  with probability p lying to its right and probability C lying between  $-t^*$  and  $t^*$ .

t distribution critical values												
	Upper-tail probability $p$											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$z^*$	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level $C$											

Table entry for p is the critical value F\* with probability p lying to its right.

F critical values											
		Degrees of freedom in the numerator									
p		1	2	3	4	5	6	7	8	9	
Degrees of freedom in the denominator	1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
		.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
		.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
		.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
		.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
	2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
		.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
		.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
		.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
		.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
	3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
		.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
		.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
		.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
		.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
	4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
		.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
		.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
		.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
		.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
	5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
		.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
		.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
		.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
		.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
	6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
		.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
		.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
.010		13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	
.001		35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69	
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	
	.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33	

