

Solutions for ICEF Olympiad 2016

General grading guidelines

Strict fairness should be used in grading.

Guidelines

1. An answer should be awarded full credit if the candidate demonstrates that masters the material and the explanation is sufficient and flawless.
2. Answers towards the correct direction, which are however insufficient may receive less than full credit and more than zero credit at the discretion of the grader.
3. Answers that are not towards the correct direction but indicate that the candidate has some limited knowledge of the material may be awarded a (small) fraction of the credit at the discretion of the grader.
4. Answers that make no sense at the discretion of the grader should be awarded zero credit.
5. Answers that are “consistently wrong” may be awarded some positive credit. That is, if a candidate makes a mistake in part (a) and then part (b) is wrong **ONLY** because of the mistake in the previous part, but the methodology is correct, then part (b) may be awarded some credit at the discretion of the grader.

Any indication of failure to master the material should result to a reduction of credit. For example, if a candidate answers the question perfectly and then attaches an additional part which is irrelevant or wrong, the grader may remove points because this indicates failure in understanding.

Any imperfection in an answer that is solely due to the use of English should not be penalized. Sometimes, candidates will use direct translation of Russian terms that do not make sense in English. However, as native Russian speakers, graders may be able to make sense of what the candidate means. No points should be removed in this case.

Appeals

Candidates have the right appeal to the decisions made by the graders according to the HSE Olympiad regulations.

The academic director of the Olympiad will decide if an appeal will be forwarded to the members of the committee for re-grading or it will be terminated for lacking merit. Appeals will be forwarded for re-grading only when one of the following occurs:

1. The candidate sufficiently justifies that the grader has made a specific mistake grading a specific part of the answer.
2. The candidate sufficiently proves that there is a problem in a specific part of the question that allows the candidate's answer to be interpreted as a (partially) correct response.
3. The candidate sufficiently proves that there is a problem in a specific part of the question such that there is no correct response.
4. The candidate sufficiently demonstrates that the given answer is also (partially) correct, even though it is not included in the answer key.

Appeals that are insufficiently justified; or do not fall under one of the above 4 cases; or are not in English; or are unclear; or are just asking for higher grades, will be terminated by the academic director of the Olympiad with the indication: "The appeal has no merit (# reason #)".

If the appeal is judged as reasonable by the academic director, a committee member will be asked to re-grade the question. In this case the candidate is possible to receive a higher or a lower or the same grade in comparison to that received in the first degree.

Question 1

(a)

If the airline serves low-income customers, hence they have a non-negative payoff, the high-income customers have a positive payoff by buying the same ticket. Hence, if the airline serves only one type of customers, it must be the high-income type.

The airline can serve the high-income customers either with business class tickets or with economy class tickets. In both cases, the optimal price of the ticket leaves the customer with 0 payoff. So, for a business class ticket: $p(1)^2=4$. For an economy class ticket $p(0)^2=1$. So $p(1)-p(0)=1$, while the difference in marginal cost is only $1/2$. Thus the airline prefers to offer the business class ticket. The profit for the airline is $10 \cdot (p(1)-1/2)=15$.

In order to serve both types of customers, the airline can for instance offer the economy class ticket only at a price of 1, so that both types have a non-negative payoff by buying the ticket. The profit for the airline would be $100 \cdot 1=100$. So the airline prefers to serve both customers.

1 point for the observation that only high-income customers can be served alone, 1 point for the observation that then the airline would offer them business class ticket, 1 point for the correct computation of profit, 2 points for any correct comparison that proves that serving both types increases the profit.

(b)

For the airline, it does not make sense to leave the low-income customers with strictly positive payoff under any ticket, since the high-income customers payoff would be higher, so the price can be increased. Hence, the airline can charge the low-income customers with a price of 1 for the economy class ticket or a price of $\sqrt{2}$ for the business class ticket. The margin is higher in the first case. Moreover, in the first case, the high-income customers can be offered a business class ticket for a price up to $\sqrt{3}$, improving the margin; in the second case, the high-income customers would rather buy an economy class ticket only for a price of 0, destroying the margin (and the low-income class would buy it too, bringing the profit to 0.)

So the airline will offer economy class tickets for $p(0)=1$ and business class tickets for $p(1)=\sqrt{3}$.

4 points for whatever correct reasoning which leads to match low-income customers with economy class tickets and high-income customers with business class tickets. Above I presented an informal but fully correct reasoning that candidates without previous training in economics could make. Of course the formal reasoning based on IR and IC constraints deserves full points too.

6 points for the correct computation of the pricing scheme.

(c)

In case of reimbursement, the airline would lose $p(0)=1$ on, in expected terms, 9 customers. The total loss is 9.

In case of discount, since the low-income passengers would enjoy the util from flying with probability 90%, the discount must bring the negative component of their payoff function down to 0,9 to still convince them to buy. So the ticket must cost $\sqrt{0,9}$. But then, high-income customers would switch to the economy class ticket, so they must be offered a discount on the business class ticket too. Since their payoff under the economy class ticket improves by 0,05 with the discount, the discount on the business class ticket must reduce the negative component of their payoff function by 0,05. So the new price has to be $\sqrt{2,9}$. The total loss from the discounts is less than $90 \cdot (0,05) + 10 \cdot (0,03) = 4,9$. So the airline will offer the discount and not the reimbursement.

1 point for the correct evaluation of reimbursement, 2 points for the correct computation of the discount on economy class tickets, 1 point for the remark that also business class tickets must be discounted, 1 point for the correct final comparison with both discounts.

Question 2

(a)

The utility maximisation problem for agent i is
$$\begin{cases} U_i = \log(c_i) + \log\left(\frac{M_i}{P}\right) \rightarrow \max_{c_i, M_i} \\ \text{s. t. } c_i + \frac{M_i}{P} \leq e_i + \frac{X_i}{P} \end{cases}$$
. The

inequality constraint can be replaced by an equality since the utility function is increasing in both c_i and M_i . The Lagrangian is $L = \log(c_i) + \log\left(\frac{M_i}{P}\right) - \lambda \left(c_i + \frac{M_i}{P} - e_i - \frac{X_i}{P}\right)$. Solving the first

order condition
$$\begin{cases} \frac{\partial L}{\partial c_i} = \frac{1}{c_i} - \lambda = 0 \\ \frac{\partial L}{\partial M_i} = \frac{P}{M_i} * \frac{1}{P} - \frac{\lambda}{P} = 0 \\ \frac{\partial L}{\partial \lambda} = -\left(c_i + \frac{M_i}{P} - e_i - \frac{X_i}{P}\right) = 0 \end{cases}$$
 leads to individual demand for the

consumption good $c_i^* = \frac{e_i + \frac{X_i}{P}}{2}$ and individual demand for nominal money $M_i^* = \frac{Pe_i + X_i}{2}$.

Grading scheme: Maximum 30% of marks for the utility maximisation problem with correctly indicated choice variables (ie c_i and M_i). It is okay if $\frac{M_i}{P}$ is used instead of M_i . It is also okay if the budget constraint is set in nominal terms instead of real terms as in the solution (ie the budget constraint can be multiplied by P). Maximum 70% of marks if the demand for money in real terms is confused with nominal terms.

(b)

Demand and supply need to be equated on each of the two markets – the market for the consumption good and the money market.

Consumption good market: $c_1^* + c_2^* = e_1 + e_2$.

Money market (in nominal terms): $M_1^* + M_2^* = X_1 + X_2$.

Grading scheme: Maximum 50% of marks for a verbal explanation. It does not matter if the money market is considered in real or nominal terms.

(c)

Solving any one of the two market clearing conditions leads to $P^* = \frac{X_1+X_2}{e_1+e_2}$. This is due to Walras law – if there are n markets and $(n - 1)$ markets clear, then the last (n^{th}) market has to clear as well. In this problem $n = 2$ with one market being the consumption good market and the other being the money market.

Grading scheme: Maximum 30% of marks if the approach to solving market clearing conditions is right but P^* is incorrect due to an error in c_i^* and/or M_i^* calculations in a). Maximum 50% of marks if P^* is correct but Walras law is ignored. Maximum 80% of marks if confirmed that the two market clearing conditions lead to the same P^* but Walras law is not mentioned explicitly.

(d)

No, the agents will not be better off as the utility level will not be affected. The price level will double: $P^{**} = 2P^* = \frac{2(X_1+X_2)}{e_1+e_2}$, which will imply unchanged c_i^* and $\frac{M_i^*}{P}$.

Grading scheme: Maximum 50% of marks for a verbal explanation without a proof that utility levels will be unchanged.

Imagine that there is also a government in this economy. The government does not produce the consumption good but can print any amount of nominal money.

(e)

The government will need to print $\Delta M = P^G g$, where P^G is the price level after the government's intervention. To calculate this price level it is necessary to modify the market clearing conditions.

Consumption good market: $c_1^* + c_2^* + g = e_1 + e_2$, where there is additional demand of g from the government.

Money market (in nominal terms): $M_1^* + M_2^* = X_1 + X_2 + \Delta M$, where there is additional supply of money ΔM from the government.

Substituting $P^G = \frac{X_1+X_2}{e_1+e_2-2g}$ from the consumption good market equilibrium or $P^G = \frac{X_1+X_2+2\Delta M}{e_1+e_2}$

in $\Delta M = P^G g$ leads to the required quantity of money printing of $\Delta M = \frac{X_1+X_2}{e_1+e_2-2g} g$.

Grading scheme: Maximum 30% of marks for a correctly defined government budget, ie $\Delta M = P^G g$. Maximum 70% of marks if one or both market clearing conditions are correctly modified but the $\Delta M = \frac{X_1+X_2}{e_1+e_2-2g} g$ is not derived. Note that there is no need to solve both market clearing conditions in this question – modifying and solving only one of the two is enough.

(f)

Obviously, higher g can be bought only with more money printing as $g = \Delta M \frac{e_1+e_2-2g}{X_1+X_2}$ as shown

in e). As $\lim_{\Delta M \rightarrow \infty} P^G = \lim_{\Delta M \rightarrow \infty} \frac{X_1+X_2+2\Delta M}{e_1+e_2} = \infty$ implies that $\lim_{P^G \rightarrow \infty} c_i^* = \lim_{P^G \rightarrow \infty} \frac{e_i + \frac{X_i}{P^G}}{2} = \frac{e_i}{2}$, the

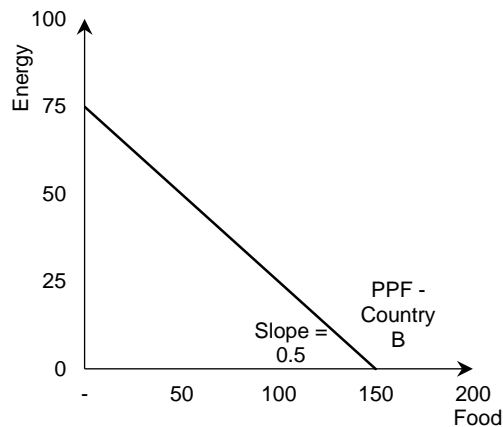
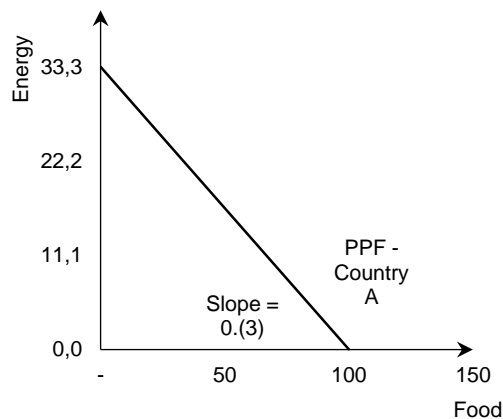
government will collect at most $e_1 + e_2 - \left(\frac{e_1}{2} + \frac{e_2}{2}\right) = \frac{1}{2}(e_1 + e_2)$, where $e_1 + e_2$ is the supply of the consumption good and $\frac{e_1}{2} + \frac{e_2}{2}$ is the demand from the agents when $P^G \rightarrow \infty$. The difference between the two is the amount purchased by the government.

Grading scheme: Zero marks if the candidate says that the government can buy $(e_1 + e_2)$. 30% marks maximum for the claim that $P^G \rightarrow \infty$ as $\Delta M \rightarrow \infty$. 80% marks for the derivation of $\lim_{P^G \rightarrow \infty} C_i^*$.

Question 3

(a)

i. The two production possibilities frontiers are below:



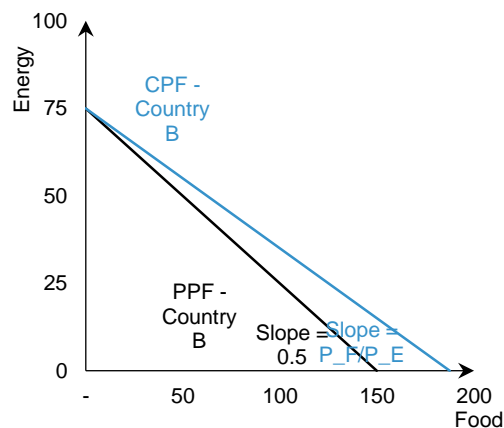
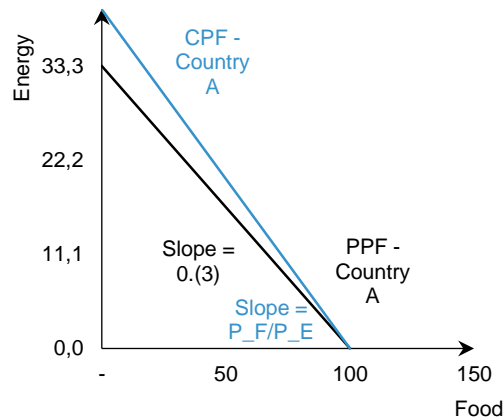
Grading scheme: It does not matter if the two production possibilities frontiers (PPFs) are plotted on one or different charts. However, the scale should be at least roughly correct in the former case, ie PPF-B should be above PPF-A and PPF-B should be flatter than PPF-A. The intercepts on Energy axis are 33.(3) and 75 for countries A and B respectively. The intercepts on Food axis are 100 and 150 for countries A and B respectively. Maximum 70% marks if any of the slopes or intercepts are missing.

ii. Country A has absolute advantage in both food and energy: one unit of labour produces 1 unit of food or 1/3 units of energy in country A while one unit of labour produces only $0.5 < 1$ units of

food or $1/4 < 1/3$ units of energy in country B. Also in order to produce one unit of food it is necessary to sacrifice $1/3$ units of energy in country A while in country B it is necessary to give up $1/2$ units of energy. This means that country A has a comparative advantage in production of food while country B has a comparative advantage in production of energy.

Grading scheme: Maximum 40% marks if only absolute advantages are stated. Maximum 60% marks if only comparative advantages are stated.

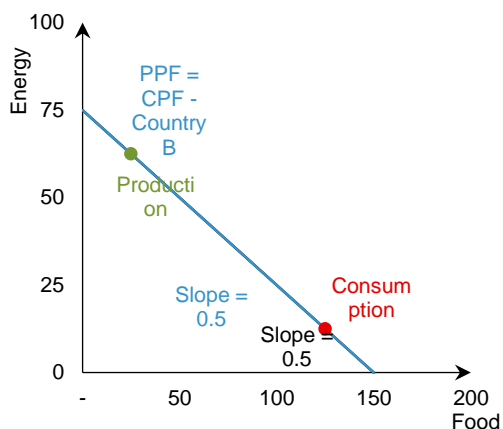
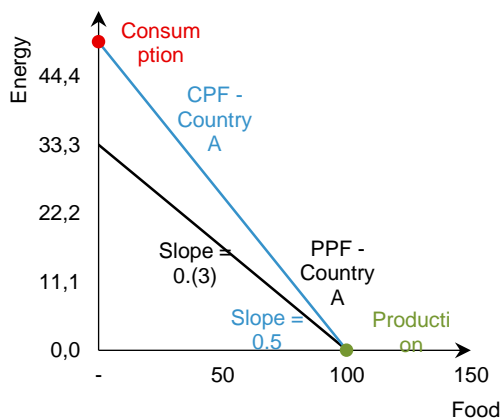
iii. The international prices of food and energy p_F and p_E have to satisfy $\frac{1}{3} \leq \frac{p_F}{p_E} \leq \frac{2}{4}$. If both signs are strict inequalities, then each country specialises in production of one good. The consumption possibilities frontiers (CPF) shift out relative the initial PPFs as shown below, which implies that each country can consume more than initially.



Grading scheme: Zero marks if product specialisations are incorrectly assigned. Maximum 50% of marks if the charts are correct but the logic with international prices is not explained.

iv. Yes, it is possible that one country B is same off. It is the larger country and it is possible that international trade leads to less than full specialisation in country B. In this case the international prices will be equal to prices in country B, ie $\frac{p_F}{p_E} = \frac{2}{4}$.

For example, country B may wish to consume 125 units of food. This demand cannot be fully covered by country A's production and B have to produce 25 units locally even if A give up all 100 units that they can produce. The charts below provide an example of production and consumption decisions that are consistent with country A being better off while country B remaining same off in presence of international trade.



Grading scheme: Maximum 40% of marks if the issue of incomplete specialisation is explained but the graphical illustration is not provided. Maximum 70% of marks if the CPFs are shown correctly but the production and consumption decisions are not discussed.

(b)

The relation is the uncovered interest parity (UIP): $1 + R_t = (1 + R_t^*) \frac{E_{t+1}^e}{E_t}$, where R_t is the interest rate in one currency, R_t^* is the interest rate in another currency, E_t is the current exchange rate and E_{t+1}^e is the expected future exchange rate.

Grading scheme: $R_t = R_t^* + \frac{E_{t+1}^e - E_t}{E_t}$ notation for the UIP is also correct. UIP with a risk premium is also correct. Maximum 30% of marks for mentioning the difference in risks without explicit reference to expected exchange rate movements. Maximum 50% of marks if future exchange rate movements are mentioned but UIP is not provided. Maximum 70% of marks if the covered interest rate parity is provided instead of UIP since forward markets may not necessarily exist.

(c)

The relation between prices of individual goods is the law of one price (LOOP): $P_i^A = E * P_i^B$, where P_i^A is the price of good i in country A, P_i^B is the price of good i in country B and E is the exchange rate. There are two relations between price levels – the absolute purchasing power parity (APPP) and the relative purchasing power parity (RPPP). The APPP is $P^A = E * P^B$, where

P^A and P^B are price levels. RPPP is $\frac{E_{t+1}-E_t}{E_t} = \pi_t^A - \pi_t^B$, where E_t is the current exchange rate, E_{t+1} is the future exchange rate and π_t^A and π_t^B are the rates of inflation in countries A and B respectively: $\pi_t^i = \frac{P_{t+1}^i - P_t^i}{P_t^i}$, P_t^i is the current price level in country i and P_{t+1}^i is the future price level in country i .

Grading scheme: Maximum 20% of marks if only LOOP is mentioned. Maximum 70% of marks if only LOOP and APPP are mentioned.

Question 4

This particular question focuses on the methodology and not on the calculations. No points should be removed for calculations as long as the candidate indicates the correct methodology.

(a)

General formula for confidence interval:

$$[\hat{\beta} - t_{cr} \cdot se(\hat{\beta}); \hat{\beta} + t_{cr} \cdot se(\hat{\beta})]$$

Critical value t_{cr} : either approximate (2) or from statistical table for t_{43} (2.0167) or for $N(0; 1)$ (1.96) --- all are considered correct.

Final answer [-0.34; -0.06] *numeric estimation not necessary for full credit*

3 points for the formula of the CI and 1 point for the substitution of the right values in the formula.

(b)

Statement of hypotheses:

H_0 : Model 1 is true OR ($\beta_{catholic} = \beta_{education} = 0$)

H_a : Model 1 is false, but Model 2 is true OR (at least one of $\beta_{catholic}$ or $\beta_{education}$ is not equal to zero)

Observed value of F-statistic

$$F_{obs} = \frac{(R_2^2 - R_1^2)/2}{(1 - R_2^2)/(47 - 4)} \approx 26.9$$

Critical value for F-statistic

$$F_{cr,2,43} = 3.21$$

Statistical conclusion: H_0 is rejected.

1 point for the statement of hypotheses, 3 points for the formula of the F-statistic, 1 point for the critical value, 1 point for describing how the statistical conclusion should be made (final conclusion is optional).

(c)

Point prediction: [+1 pt]

$$\hat{y}_{n+1} = 60.304 + 50 \cdot 0.194 = 70$$

Estimation of standard error of regression $\hat{\sigma}^2$:

$$\hat{\sigma}^2 = \frac{\text{RSS}}{n - k} = \frac{6283}{47 - 2} = 139.6$$

Formula for estimate of variance of forecast:

$$\text{se}^2(\hat{\beta}_1 + \hat{\beta}_2 \cdot 50) = \text{se}^2(\hat{\beta}_1) + 50^2 \text{se}^2(\hat{\beta}_2) + 2 \cdot 50 \cdot \hat{\text{Cov}}(\hat{\beta}_1, \hat{\beta}_2) \approx 2.97$$

Formula for estimate of forecast variance

$$\text{se}^2(\hat{y}_{n+1} - y_{n+1}) = \text{se}^2(\hat{\beta}_1 + \hat{\beta}_2 \cdot 50) + \hat{\sigma}^2 = 2.97 + 139.6 \approx 146$$

Formula for predictive interval

$$[\hat{y}_{n+1} - t_{cr} \text{se}(\hat{y}_{n+1} - y_{n+1}); \hat{y}_{n+1} + t_{cr} \text{se}(\hat{y}_{n+1} - y_{n+1})]$$

Critical value of t-statistic: either approximate (2) or from statistical table for t_{45} (2.0141) or for $N(0; 1)$ (1.96) - all are considered correct

Final answer: [46; 94] *numeric estimation not necessary for full credit*

1 point for the statement of the point prediction, 2 points for the statement of the formula of the standard error formula and substitution of values, 2 points for the formula for estimate of variance of forecast and substitution of values, 2 points for formula and substitution for estimate of forecast variance $\text{se}^2(\hat{y}_{n+1} - y_{n+1})$, 2 points for the explanation of how the CI can be derived.

Question 5

(a)

This purely a basic valuation exercise. We need projections for cash flows and corresponding required return. Before any changes company will generate \$10 forever, generate free cash flow to equity (no growth strategy), and should pay dividends \$10 per share in dividends. The first dividend will be paid tomorrow (not at end of the first year as it is usually assumed). Required return on equity according to CAPM is

$$R_e = 0,04 + 1,2(0,14 - 0,04) = 0,16$$

Discounting expected dividends gives us the following:

$$\text{Stock value of ABC Value} = 10/0,16 + 10 = 72,5$$

Marking scheme:

Projected dividend stream (1.5 pts)

Required return on ABC stock (1.5 pts)

Fair value of ABC stock (2 pts)

(b)

We infer we can derive required return on operating asset type B from companies that already exploit that type of business. A usual approach to that is to find a comparable company and try

to work out betas (or returns) for assets and invested capital. All the risks generated by assets are spread among participating investors (debtholders and equityholders).

Sum of value weighted asset betas = Sum of value weighted capital betas

A participant should be able to identify that except operating assets, XYZ has a financial asset – PVTS (Present value of tax shield). It is associated with interest tax savings from debt. XYZ interest tax savings are perpetual and its present value (given the assumption of the level of risk) is $D \cdot T_c = 1000 \cdot 0,2 = 200$.

Finally, one can infer that XYZ has 3 assets (operating - type B, type C and PVTS – financial asset) that are financed with riskless debt and equity. Value balance sheet will look like

Asset type	Value	Betas	Capital	Value	Beta
Type B	600	Unknown	Debt	1000	0
Type C	1200	1	Equity	1000	1,44
PCTS	200	0			

One can find out that beta of Operating Asset type B is 0,4

Finally, plug into CAPM and get required return $R_b = 0,04 + 0,4(0,14 - 0,04) = 0,08$

Marking scheme:

Identified financial PVTS asset (1.5 pts)

Formulated the correct logic to answer the question (1.5 pts)

Determined beta of type B operating asset (or required return) (2 pts)

(c)

Since we don't have any imperfections that might affect the fair value investor will expect stock value to change based on economic profit generated by investment in type B. One should be able to tell whether investment in type B has positive, negative or zero NPV. One needs to answer the question of whether asset B is expected to generate enough returns to compensate the investment risks. It is clearly stated in the problem that type B are expected to generate 12%. The required return on type B comes from the previous problem and equals 8%. Finally, one should be able to conclude that this is positive NPV investment. Therefore, value of the stock is expected to increase. This is the only reason for ABC stock to change.

Marking scheme:

Discussed potential reasons for value change (1 pts)

Stated that the only reason is NPV of investment and determined whether NPV is negative or positive (3 pts)

(d)

There are a number of ways to answer that question. One is expected to show his/her knowledge skills to determine cash flows, pick up corresponding discount rates and determine the ABC's stock fair value

In the problem it is given that management offers to invest part of current free cash flows in type B business. All future investment and payout decisions are not altered by Dec 31 investment idea. An investor will trade part of his current dividend for future cash flows. To value ABC equity after investment decision is taken and made publicly available, we need to analyze 2 sources of cash flows and risks. One is expected cash flows from existing type A asset. The other is cash flows from new type B asset. In per share terms:

Asset type	Dividends tomorrow	Dividends expected in a year	Required return
Type A	6=10-4	10 and the same thereafter	0,16
Type B		0,48 = 4*0,12 and the same thereafter	0,08

Since ABC has no debt and no tax savings, shareholder will care only about operating assets. Finally, value of ABC stock will be determined by discounted values of cash flows of two types of businesses:

Value from type A = $6 + 10/0,16 = \$68,5$

Value from type B = $0,48/0,08 = \$6$

ABC stock fair value = $68,5 + 6 = \$74,5$

We see stock value increased from 72,5 to 74,5. This is clearly an indication of positive NPV investment.

Marking scheme:

Projected changes in dividend stream (3 pts)

Determined new stock value (3 pts)

End of file