Solutions for demo Olympiad

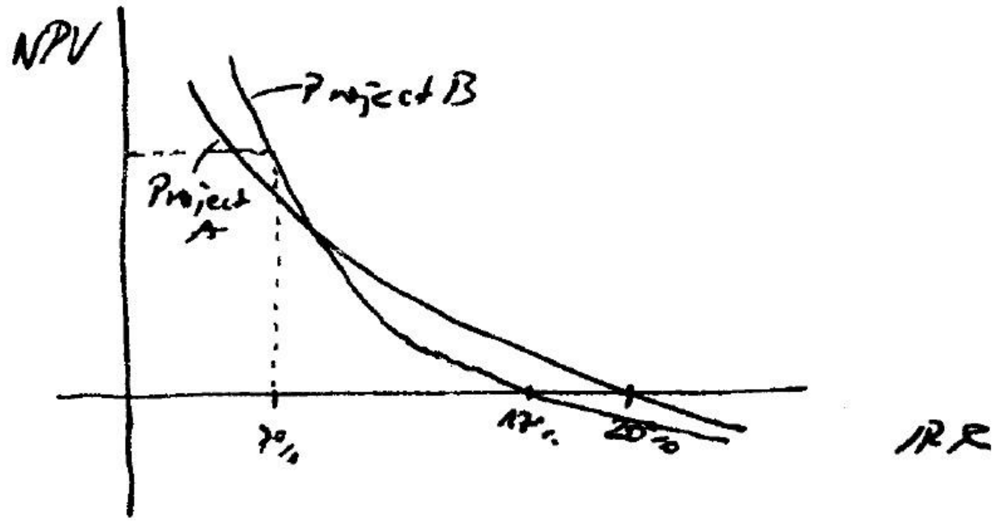
Question 1 [20pts]

a) [3p.]
Project A: $-10 + \frac{2}{IRR_A} = 0 \quad \rightarrow \quad IRR_A = 20\%$
Project B: $-10 + \frac{1.5}{IRR_B - 2\%} = 0 \quad \rightarrow \quad IRR_B = 17\%$

b) [4p.]
Our firm: $E[R] = 3\% + 0.8 * (E[R_M] - 3\%)$
The other firm: $10\% = 3\% + 1.4 * (E[R_M] - 3\%) \quad \rightarrow \quad E[R_M] = 8\%$
Therefore, $E[R] = 3\% + 0.8 * (8\% - 3\%) = 7\%$.

c) [3p.]
$NPV_A = -10 + \frac{2}{0.07} = 18.57$ (mln dollars)
$NPV_B = -10 + \frac{1.5}{0.07 - 0.02} = 20$ (mln dollars)

d) [5p.: 2p. for the investment rules and decisions (if they are formulated for general projects such as NPV>0 and IRR>required return, 1 p. can be given; 1p. for the graph, 2p. for the reasoning. Students might cite another reason – the different scale of the projects. This is, when you double all cash flows of a project, the NPV will double but IRR will stay the same. This cannot be the whole story here since the project with the (initially) smaller cash flows has the higher NPV. 1 point can be given for this answer if timing of cash flows is not mentioned.]
NPV rule for mutually exclusive projects: Invest in the project with the higher NPV.
Decision: invest in project B.
IRR rule for mutually exclusive projects: invest in the project with the higher IRR.
Decision: invest in project A.



The reason for the different recommendations emanating from the two rules is that the timing of cash flows is different in the two projects. The NPV and the IRR rules make different assumptions about the reinvestment of cash flows: the NPV rule assumes that they can be reinvested at the market-determined cost of capital, while the IRR rule assumes that they can be reinvested at the IRR for each project. Arguably, the first assumption is better: Since the risk of both projects is the same, the reinvestment rates cannot be different.

e) [5p.: 1p. for computing the incremental cash flows and 2p. for the incremental IRR; 2p. for the investment rule and decision.]
Incremental IRR. The incremental cash flows are $2 - 1.5 = 0.5$ in the first year, $2 - 1.5 * 1.02 = 0.47$ in the second year, $2 - 1.5 * 1.02^2 = 0.4394$ in the third year, etc.
The IRR of the incremental cash flows can be computed by

$$\frac{2}{IRR_{inc}} - \frac{1.5}{IRR_{inc} - 2\%} = 0 \quad \rightarrow \quad IRR_{inc} = 8\%$$

Investment rule: At rates of required return smaller than 8%, project B is preferable. At rates higher than 8%, project A is preferable. In our case the expected return is 7% (part b)), so we would recommend to invest in project B.

Question 2 [20pts]

a) [4p.] $\pi_A=(p_A-c)q_A$; d π_A /d p_A = q_A + (p_A-c) d q_A /d p_A = s-b(p_A-p_B)- b(p_A-c) =0. FOC for Boeing is symmetric. Best response of Airbus: p_A=1/2 s/b+1/2$(p_B$+c). BR for Boeing is symmetric. Equilibrium: $p_A=p_B$=s/b+c; $q_A=q_B$=s; $\pi_A=\pi_B=s^2/b$.
b) [6p.] d π_A /d p_A = s-b$(p_A-p_B+t_A-t_B)$- b(p_A-c) =0, symmetric for B. Equilibrium: p_A =s/b+c-1/3(t_A-t_B); q_A=s-b/3(t_A-t_B); π_A=(s/b-1/3(t_A-t_B))(s-b/3(t_A-t_B)); $R_A=t_A$(s-b/3(t_A-t_B)). The answers for Boeing are symmetric.

c) [6p.] Europe maximizes $\pi_A+R_A=(s/b+2/3t_A+1/3t_B)(s-b/3(t_A-t_B))$ over t_A; best response is $t_A=1/4t_B+3/4s/b$, likewise for Boeing. Equilibrium: $t_A=t_B=s/b$. Profits are unchanged from (a).

d) [4p.] Yes, profit+tax revenue increases due to inelastic nature of demand: while both prices are increased by taxation, the aggregate demanded quantity does not change.

Question 3 [20pts]

a) **[9 points]**

$$L=\prod_{i=1}^{10}\frac{e^{\alpha+\beta}}{1+e^{\alpha+\beta}}\prod_{i=1}^{35}\frac{1}{1+e^{\alpha+\beta}}\prod_{i=1}^{15}\frac{e^{\alpha}}{1+e^{\alpha}}\prod_{i=1}^{40}\frac{1}{1+e^{\alpha+\beta}}=\frac{e^{25\alpha}}{(1+e^{\alpha})^{100}}$$

$l=lnL=25\alpha+10\beta-45\ln\left(1+e^{\alpha+\beta}\right)-35\ln(1+e^{\alpha})$ [3 points]

$\frac{\partial l}{\partial\alpha}=25-45\frac{e^{\alpha+\beta}}{1+e^{\alpha+\beta}}-55\frac{e^{\alpha}}{1+e^{\alpha}}$ [2 points]

$\frac{\partial l}{\partial\beta}=10-45\frac{e^{\alpha+\beta}}{1+e^{\alpha+\beta}}$ [2 points]

$\begin{cases}\frac{\partial l}{\partial\alpha}=0\\ \frac{\partial l}{\partial\beta}=0\end{cases}\Leftrightarrow\begin{cases}\hat{\alpha}=ln\frac{3}{8}\\ \hat{\beta}=ln\frac{16}{21}\end{cases}$ [2 points]

b) **[4 points]** The probability that a male is a smoker: Pr(y=1|x=1)= $\frac{e^{\hat{\alpha}+\hat{\beta}}}{1+e^{\hat{\alpha}+\hat{\beta}}}=\frac{2}{9}$

[2 points]

The probability that a female is a smoker: Pr(y=1|x=0)= $\frac{e^{\hat{\alpha}}}{1+e^{\hat{\alpha}}}=\frac{3}{11}$ [2 points]

c) **[7 points]** $L(\alpha)=\prod_{i=1}^{10+15}\frac{e^{\alpha}}{1+e^{\alpha}}\prod_{i=1}^{35+40}\frac{1}{1+e^{\alpha}}=\frac{e^{25\alpha}}{(1+e^{\alpha})^{100}}$ [2 point]

$\frac{\partial L(\alpha)}{\partial\alpha}=\frac{25e^{25\alpha}(1-3e^{\alpha})}{(1+e^{\alpha})^{101}}=0\Leftrightarrow e^{\alpha}=3\Leftrightarrow\alpha=ln\frac{1}{3}$ [3 points]

Thus Pr(y=1)= $\frac{e^{\alpha}}{1+e^{\alpha}}=\frac{1}{4}$ [2 points]

Question 4 [20pts]

Solution

a) [9p.] The OBEC countries announce in advance the quantity they are going to produce in the next period. Thus we have Stackelberg market structure with the leader represented by OBEC countries and newcomer as the follower. Lets assume that the total quantity produced by OBEC countries is given by Q_1, while newcomer produces q_2 and there are n firms on the market. To solve this problem we should start from the newcomer problem:

$$\max_{q_2} (a - b(Q_1 + q_2) - c)q_2$$

The FOC is

$$\frac{\partial \pi_2}{\partial q_2} = a - bQ_1 - 2bq_2 - c = 0$$

Solving we get:

$$q_2^*(Q_1) = \frac{a - bQ_1 - c}{2b}$$

The problem of the leader is the following:

$$\max_{Q_1} (a - bQ_1 - b\frac{a - bQ_1 - c}{2b} - c)Q_1$$

The FOC is

$$\frac{\partial \pi_1}{\partial Q_1} = a - 2bQ_1 - \frac{a - 2bQ_1 - c}{2} - c = 0$$

Solving we get:

$$Q_1^* = \frac{a - c}{2b}$$

Thus each OBEC country produces $q_1^* = \frac{Q_1^*}{n-1} = \frac{a-c}{2(n-1)b}$. Substituting it to $q_2^*(Q_1)$ we find:

$$q_2^* = \frac{a - c}{4b}$$

Now we can find bananas price which is $P^* = a - b\frac{a-c}{2b} - b\frac{a-c}{4b} = \frac{a+3c}{4}$. The profits of each OBEC country is

$$\pi_1^* = \left(\frac{a+3c}{4} - c\right)\frac{a-c}{2(n-1)b} = \frac{(a-c)^2}{8(n-1)b}$$

The profit of newcomer is:

$$\pi_2^* = \left(\frac{a+3c}{4} - c\right)\frac{a-c}{4b} = \frac{(a-c)^2}{16b}$$

Marking scheme:

- 1 point for understanding that we should use the Stackelberg framework.
- 3 points for solving the newcomer problem for quantity.
- 3 points for solving the leader problem for quantity.
- 2 points for solving for price and profits.

b) [6p.] In this case all countries decide simultaneously about the quantity they are going to produce and we have Cournot oligopolistic model. The profit maximization problem of the first country is:

$$\max_{q_1}(a - b\textstyle\sum_{k=1}^{n} q_k - c)q_1$$

The FOC is

$$\frac{\partial\pi}{\partial q_1} = a - 2bq_1 - b\textstyle\sum_{k\neq1} q_k - c = 0$$

Note that the same FOC condition would be for other $n-1$ countries. Substituting $q_C = q_i$ we get $a - 2bq_C - b(n-1)q_C - c = 0$. The solution is:

$$q_C^* = \frac{a-c}{(n+1)b},\ P_C^* = \frac{a+nc}{n+1}$$

The profit of each country is:

$$\pi_C^* = \frac{(a-c)^2}{b(n+1)^2}$$

Countries would prefer to stay in OBEC if profits in the case of Stackelberg are higher than in Cournot:

$$\frac{(a-c)^2}{8(n-1)b} > \frac{(a-c)^2}{b(n+1)^2}$$

$$(n+1)^2 > 8(n-1)$$

$$n^2 - 6n + 9 > 0$$

Solving for roots we get just one root $n = 3$. Thus countries are indifferent whether to keep OBEC or act as independent producers if $n = 3$, but for all other n OBEC is preferable.

Marking scheme:

- 1 point for understanding that we should use the Cournot framework.
- 3 points for solving Cournot problem for quantity, price and profits.
- 2 points for showing that acting together as OBEC countries is always better.

c) [5p.] The newcomer will prefer the world without OBEC when it has higher profits in Cournot model, which happens when $\pi_C^* > \pi_2^*$:

$$\frac{(a-c)^2}{b(n+1)^2} > \frac{(a-c)^2}{16b}$$

The inequality reduces to the condition $n < 3$. Thus if there are less than two countries in OBEC the newcomer would prefer the world without OBEC. However, if OBEC consists of more than two countries the newcomer prefers the world with OBEC. The explanation is quite easy. The more countries belong to OBEC, the more efficient the organization in reduction of bananas production. One of the drawbacks of our model is that we do not take into account that the more countries are in OBEC the harder it is to coordinate on some level of output.

Marking scheme:

- 3 points for finding the condition.
- 2 points for explaining the result.

Question 5 [20pts]

a) [5p.] The marginal rate of transformation is 1, so the wage rate is 1.

b) [6p.] Preferences are identical across periods and goods, hence at the optimal the allocation will be the same across periods. the marginal rate of substitution between consumption and labour equals the marginal rate of transformation which is 1. As there is a representative consumer, $c=l$ hence the allocation is (1/2, 1/2) for both labour an consumption.

c) [9p.] Welfare is maximised if consumption is the same in both periods. So better to raise $g(0)/2$ in the first period through taxes and $g(0)/2$ in the second period. To see this, budget constraint of household is

$$c(0)+b(0)+\tau(0)w(0)l(0)\leq w(0)l(0)$$

and

$$c(1)-b(0)(1+r(0))+\tau(1)w(1)l(1)\leq w(1)l(1).$$

Present value budget constraint gives

$$c(0)+\frac{c(1)}{1+r(0)}\leq\big(1-\tau(0)\big)w(0)l(0)+\frac{\big(1-\tau(1)\big)w(1)l(1)}{1+r(0)}.$$

Therefore government receives

$$g(0)=\tau(0)w(0)l(0)+\frac{\tau(1)w(1)l(1)}{1+r(0)}.$$

Welfare is maximized if consumption and labour are identical across periods. So we check whether this is feasible. Assume $c(0)=c(1)=c$ and $l(0)=l(1)=l$. Then $r(0)=0$, $\frac{c}{l}=\big(1-\tau(0)\big)+\big(1-\tau(1)\big)$ and

$$\frac{g(0)}{\tau(0)+\tau(1)}=l.$$

First order condition gives

$$\frac{1}{1-l}=\frac{1-\tau(0)}{c}=\frac{1-\tau(1)}{c}.$$

Hence $\tau(0)=\tau(1)=\tau$. Therefore $\frac{g(0)}{2\tau}=l$ and $c=(1-\tau)\frac{2\tau-g(0)}{2\tau}$. Using first order condition

$$c=(1-\tau)l$$

(1)

$$(1-\tau)\frac{2\tau-g(0)}{2\tau}=(1-\tau)\frac{g(0)}{2\tau}$$

(2)

$$g(0)=\tau$$

(3)

Therefore $l(0)=l(1)=1/2$, $\tau(0)l(0)=\tau(1)l(1)=1/2g(0)$ and $c(0)=c(1)=(1-g(0))1/2$.

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