**Olympiad for students and graduates – 2017**

**Demonstrative version for the area**
**«Mathematics»**

**You have 240 min to complete this task.**

***Each problem costs 20 points, if the sum exceeds 100, the result is equal to 100 points.***

# I. COMMON PART

*Solutions of the problems in this section should be written in Russian or in English.*

**1.** Does the following series converge

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{n+1}{n}}}?$$

**2.** Let $G$ be a finite group, and $\alpha : G \to G$ an automorphism of $G$ such that $\alpha(x) = x$ only if $x$ is the identity. Prove that every element of the group $G$ can be represented in the form $x^{-1}\alpha(x)$, where $x \in G$.

**3.** In the Euclidean space $\mathbb{R}^3$, an ellipsoid with semi-principal axes of lengths $a$, $b$, $c$ is given. A rectangular parallelepiped is circumscribed around it so that the ellipsoid is tangent to all faces of the parallelepiped. Compute the length of the principle diagonal in the parallelepiped.

**4.** Solve the initial value problem $u_t = -u^2 u_x$, $u(x, 0) = \cos x$. Find the maximal value of $T$ such that a non-singular solution exists on the set $t \in [0, T)$, $x \in \mathbb{R}$.

# II. SPECIAL PART

*Solutions of the problems in this section should be written in* ***English****.*

**1.** Let $n$ be the number of ordered triples $(A_1, A_2, A_3)$ consisting of sets $A_1$, $A_2$, $A_3$ such that

$$A_1 \cup A_2 \cup A_3 = \{1,\, 2,\, 3,\, 4,\, 5,\, 6,\, 7,\, 8,\, 9,\, 10\},$$
$$A_1 \cap A_2 \cap A_3 = \emptyset.$$

Find the prime factorization of $n$.

**2.** Let $V$ be a finite dimensional vector space over the field of real numbers. The sum $A + B$ of sets $A, B \subset V$ is defined as the set of all vectors of the form $a + b$, where $a \in A$, $b \in B$. For any $\lambda \in \mathbb{R}$, the set $\lambda A$ is by definition the set of all vectors of the form $\lambda a$, where $a \in A$. Prove that an open set $A$ satisfies the equality $A + A = 2A$ if and only if $A$ is convex.