# Answers for ICEF Olympiad 2017

## General grading guidelines

These are 5 original problems created by ICEF academics for the purpose of the Olympiad. Each problem has been created by a specialist in the area, edited by the academic director of the Olympiad and then reviewed independently by another specialist. These are open ended problems. The answers will be marked blindly at the discretion of the grading team in a manner uniform for all candidates.

**Guidelines**

1. Full credit should be reserved for answers that demonstrate mastering the material and the explanation is sufficient and flawless at the discretion of the grader.
2. Partial credit may be given to answers towards the correct direction, which are however insufficient at the discretion of the grader.
3. Some positive credit may be given to answers not towards the correct direction, which however indicate that the candidate has some limited knowledge on the subject at the discretion of the grader.
4. Zero credit should be given to answers that are wrong, irrelevant or make no sense at the discretion of the grader.

Any indication of failure to master the material should result to a reduction of credit. For example, if a candidate answers the question perfectly and then attaches an additional part which is irrelevant or wrong, the grader may remove points because the redundancy indicates failure in understanding.

**Appeals**

Candidates have the right to appeal to the decisions made by the graders according to the HSE Olympiad regulations.

The academic director of the Olympiad will decide if an appeal will be forwarded to the members of the committee for re-grading or it will be terminated as lacking merit. Appeals will be forwarded for re-grading only when one of the following occurs:

1. <u>The candidate sufficiently justifies</u> that the grader has made a specific mistake grading a specific part of the answer.
2. <u>The candidate sufficiently proves</u> that there is a misspecification in some part of the question that allows the candidate's answer to be interpreted as a (partially) correct response.
3. <u>The candidate sufficiently proves</u> that there is a mistake in a specific part of the question such that there is no correct response.

4. The candidate sufficiently demonstrates that the given answer is also (partially) correct, even though it is not included in the answer key provided by ICEF.

Appeals that are insufficiently justified; or do not fall under one of the above 4 cases; or are not in English; or are unclear; or are just asking for higher grades, will be terminated by the academic director of the Olympiad with the indication: “The appeal has no merit (# reason #)”.

If the appeal is judged as reasonable by the academic director, a committee member will be asked to re-grade the question. In this case the candidate is possible to receive a higher or a lower or the same grade in comparison to that received in the first degree.

**Answer for problem 1**

a) The budged constraint is easily derived. It is simply the amount of good x multiplied by its price plus the amount of good y multiplied by its price. This sum cannot exceed the wealth of Gerhard. Thus we have:

$$xP_x + yP_y \leq W$$

(2 points for the BC)

So, the maximization problem is:

$$\max_{x,y\geq 0} \min(\alpha x, \beta y)$$
$$\text{s.t.:} \quad xP_x + yP_y \leq W$$

(2 points for the proper set up)

To derive the optimal bundle, first, it is necessary to understand that this is a Leontief utility function and that by that token just taking FOC's will not get you very far. Instead, you need to realize that at any optimal point:

$$\alpha x = \beta y$$

(2 points for the realization)

must hold. This can be plugged into the Budged constraint to yield:

$$xP_x + \frac{\alpha x}{\beta} P_y = W$$

This can be rearranged to yield:

$$\beta x P_x + \alpha x P_y = \beta W$$

And further to yield the demand for good x given wealth W:

$$x = \frac{\beta W}{\beta P_x + \alpha P_y}$$

A symmetrical procedure gives us the demand for good y given wealth W:

$$y = \frac{\alpha W}{\beta P_x + \alpha P_y}$$

(1 points for proper derivation, 1 point for the actual demands)

This concludes part a.

b) Here, we simply want to see the graph showing the Leontief utility indifference curves with their characteristic kink.

(1 point)

We also want the BC that intersects the x-axis at $\frac{W}{P_x}$ and the y-axis at $\frac{W}{P_y}$.

(1 point)

Finally, the student is asked to point out the bundle of x and y that provides the highest utility.

(2 points)

c) Simply recall the definition of Income elasticity of demand: $\frac{\partial x}{\partial W}\frac{W}{x}$ which is the percentage change of consumption of good x when Income (or in our case wealth) changes by one percent. In our case this is:

$$\frac{\beta}{\beta P_x+\alpha P_y}\frac{W}{\frac{\beta W}{\beta P_x+\alpha P_y}}=1$$

for good x and:

$$\frac{\alpha}{\beta P_x+\alpha P_y}\frac{W}{\frac{\alpha W}{\beta P_x+\alpha P_y}}=1$$

for good y. This tells us they **are both normal goods** since their consumption increases as wealth increases. Further, it will increase in fixed proportion and thus consumption will grow at the same rate as the disposable wealth.

(1 point for elasticities and reasoning each = 2 points)

Of course for the own price elasticities it is much the same. The definition is: $\frac{\partial x}{\partial P_x}\frac{P_x}{x}$, so it will be:

$$\frac{-\beta^2}{(\beta P_x+\alpha P_y)^2}\frac{P_x}{\frac{\beta W}{\beta P_x+\alpha P_y}}=\frac{-\beta P_x}{(\beta P_x+\alpha P_y)}$$

and:

$$\frac{-\alpha^2}{(\beta P_x+\alpha P_y)^2}\frac{P_y}{\frac{\alpha W}{\beta P_x+\alpha P_y}}=\frac{-\alpha P_y}{(\beta P_x+\alpha P_y)}$$

This tells **us x and y are both ordinary goods** because their own price elasticity is negative which means their demand drops as their price increases.

(0.5 point for elasticities and reasoning each = 1 point)

Finally, let us recall the definition of cross price elasticities: $\frac{\partial x}{\partial P_y}\frac{P_y}{x}$ where x and y are not the same good. So, for good x we get:

$$\frac{-\alpha\beta}{(\beta P_x+\alpha P_y)^2}\frac{P_y}{\frac{\beta W}{\beta P_x+\alpha P_y}}=\frac{-\alpha P_y}{(\beta P_x+\alpha P_y)}$$

And for good y, we get:

$$\frac{-\alpha\beta}{(\beta P_x+\alpha P_y)^2}\frac{P_x}{\frac{\alpha W}{\beta P_x+\alpha P_y}}=\frac{-\beta P_x}{(\beta P_x+\alpha P_y)}$$

We conclude **good x and y are complements** since their cross-price elasticities are negative, indicating that an increase in the price of one good decreases the demand for the other good as well.

(0.5 point for elasticities and reasoning each = 1 point)

d) Now, the previous exercise should have already given the student a hint that it probably does not matter, since the income elasticities are both unitary and the cross price elasticities are in fact exactly the same as the own price elasticities, just switched around. With this in mind, let us derive the size of the tax on good x that is necessary to raise T. First, realize that then the demand for good x is:

$$x = \frac{\beta W}{\beta(P_x + t_x) + \alpha P_y}$$

For good y it would be:

$$y = \frac{\alpha W}{\beta P_x + \alpha(P_y + t_y)}$$

(0.5 point for writing down the modified demand with the tax rates)

Then we need to understand that the tax $t_x$ is multiplied by the amount of x consumed to get the total tax revenue that needs to be equal to T.

$$T = t_x \frac{\beta W}{\beta(P_x+t_x)+\alpha P_y}$$

This can be solved for the tax, yielding:

$$t_x = \frac{T(\beta P_x + \alpha P_y)}{\beta(W - T)}$$

The exact same logic gives you the tax for good y:

$$t_y = \frac{T(\beta P_x + \alpha P_y)}{\alpha(W - T)}$$

(0.5 for deriving either tax and 0.5 for realizing that it must be the same for the other = 1 point)

Once we plug in the tax rate into the demand functions for x and y, we first realize that the tax for x reduces demand for x and y in equal proportions. This comes from the cross price elasticity being the same as the own price elasticity and from the fact that these two goods are perfect complements.

$$x = \frac{\beta W}{\beta(P_x+\frac{T(\beta P_x+\alpha P_y)}{\beta(W-T)})+\alpha P_y} = \frac{\beta W}{\beta P_x+\frac{T(\beta P_x+\alpha P_y)}{W-T}+\alpha P_y}$$

$$y = \frac{\alpha W}{\beta(P_x+\frac{T(\beta P_x+\alpha P_y)}{\beta(W-T)})+\alpha P_y} = \frac{\alpha W}{\beta P_x+\frac{T(\beta P_x+\alpha P_y)}{W-T}+\alpha P_y}$$

(0.5 point for plugging the tax rate for x correctly into the demand for x and y)

Further, you will realize that plugging in the tax rate for good y that raises T in revenue, gives you the exact same effect on good y as the tax rate for good x. Again, this is due to the perfect complementarity of the two goods.

$$y = \frac{\alpha W}{\beta P_x+\alpha(P_y+\frac{T(\beta P_x+\alpha P_y)}{\alpha(W-T)})} = \frac{\alpha W}{\beta P_x+\frac{T(\beta P_x+\alpha P_y)}{W-T}+\alpha P_y}$$

$$x = \frac{\beta W}{\beta P_x+\alpha(P_y+\frac{T(\beta P_x+\alpha P_y)}{\alpha(W-T)})} = \frac{\beta W}{\beta P_x+\frac{T(\beta P_x+\alpha P_y)}{W-T}+\alpha P_y}$$

(0.5 point for plugging the tax rate for y correctly into the demand for x and y)
With this, the students should realize that it does not matter which good is taxed since either tax has the exact same effect on both(!) goods because they are perfect complements and the taxes are therefore equivalent to each other as they have only an income effect (substitution is per definition not possible). Any combination of the two taxes would yield the same outcome.
(1 point for the realization it does not matter which good is taxed, 0.5 point for the reasoning = 1.5points)

**Answers for problem 2**

a) What should be the sign of parameter $\beta$ and why? [2p]

**Solution**: $\beta$ is positive because positive output gap $(y-\bar{y})$ puts upward pressure on prices, which means higher inflation.

**Grading scheme**: Zero points with no explanation. 50% of points for unclear explanations.

b) What inflation rate will the central bank choose? [4p]

**Solution**: The central bank will minimise its loss subject to the Phillips curve:

$$\begin{cases} L = (\pi - \pi^*)^2 + \lambda(y - y^*)^2 \rightarrow \min\limits_{\pi} \\ s.t. \pi = \alpha + \beta(y - \bar{y}) \end{cases}$$

$y$ from the Phillips curve can be substituted in $L$ reducing the problem to unconstrained minimisation of $L = (\pi - \pi^*)^2 + \lambda\left(\frac{\pi-\alpha}{\beta} + \bar{y} - y^*\right)^2$ with respect to $\pi$, which leads to $\pi = \frac{\beta^2\pi^* + \lambda\alpha + \lambda\beta(y^* - \bar{y})}{\beta^2 + \lambda}$.

**Grading scheme**: 50% of points for correctly stated central bank's problem. 70% of points if the final answer is incorrect due to arithmetical errors.

c) Assume that $\alpha = \pi^*$ in the Phillips curve. Explain if the actual inflation rate coincides with the inflation target. [3p]

**Solution**: Substituting $\alpha = \pi^*$ in the equation for the actual inflation rate derived in (b) yields $\pi = \pi^* + \frac{\lambda\beta(y^* - \bar{y})}{\beta^2 + \lambda}$. It does is not coincide with the inflation target $\pi^*$ if $y^* \neq \bar{y}$. Specifically, if $y^* > \bar{y}$, the central bank pushes inflation above $\pi^*$ to raise output above its natural level. $\frac{\lambda\beta(y^* - \bar{y})}{\beta^2 + \lambda}$ is the inflation bias, which depends on the degree of inflation aversion of the central bank ($\lambda$)and the slope of the Phillips curve ($\beta$).

**Grading scheme**: 50% of points if the reasons for the inflation deviations from $\pi^*$ are not explained.

d) If the central bank is *myopic*, i.e. cares only about the current period, what is the rational expectation of expected inflation? [4p]

**Solution**: The central bank will minimise one-period loss subject to the Phillips curve:

$$\begin{cases} L_t = (\pi_t - \pi^*)^2 + \lambda\left(y_t - y^*\right)^2 \rightarrow \min\limits_{\pi_t} \\ s.t. \pi_t = \pi_t^e + \beta\left(y_t - \bar{y}\right) + \varepsilon_t \end{cases}$$

$y_t$ from the Phillips curve can be substituted in $L_t$ reducing the problem to unconstrained minimisation of $L_t = (\pi_t - \pi^*)^2 + \lambda\left(\bar{y} + \frac{\pi_t - \pi_t^e - \varepsilon_t}{\beta} - y^*\right)^2$ with respect to $\pi_t$. The first order condition (FOC) is $2(\pi_t - \pi^*) + 2\frac{\lambda}{\beta}\left(\bar{y} + \frac{\pi_t - \pi_t^e - \varepsilon_t}{\beta} - y^*\right) = 0$. Since the public expectations are rational/model-consistent, they will be formed at time $(t-1)$ based on the FOC for the central bank's problem at time $t$: $\mathrm{E}_{t-1}\left(2(\pi_t - \pi^*) + 2\frac{\lambda}{\beta}\left(\bar{y} + \frac{\pi_t - \pi_t^e - \varepsilon_t}{\beta} - y^*\right)\right) = 0$. Noting that $\mathrm{E}_{t-1}(\varepsilon_t) = 0$, $\mathrm{E}_{t-1}\pi_t = \pi_t^e$ and $\mathrm{E}_{t-1}\pi_t^e = \pi_t^e$ due to the law of iterated expectations, the equation reduces to $\pi_t^e = \pi^* + \frac{\lambda}{\beta}(y^* - \bar{y})$.

**Grading scheme**: 25% of points for correctly stated central bank's problem. 50% of points for applying the expectations operator as of time $(t-1)$ to the FOC as of time $t$. 75% of points if the final answer is incorrect due to arithmetical errors.

e) Given your result in (d), what is the actual inflation rate? [4p]

**Solution**: $\pi_t^e$ from (d) should be substituted back in the FOC, which leads to $\pi_t = \pi^* + \frac{\lambda}{\beta}(y^* - \bar{y}) + \frac{\lambda}{\beta^2+\lambda}\varepsilon_t$.

**Grading scheme**: 50% for the attempt to substitute the result from (d) back in the FOC. 75% of points if the final answer is incorrect due to arithmetical errors other than in (d).

f) How do shocks $\varepsilon_t$ affect the central bank's choice of inflation rate? [3p]

**Solution**: When $\varepsilon_t > 0$, there is an adverse shock and the central bank reacts by increasing the rate of inflation to reduce the negative impact of the shock on the level of output. Favourable shocks $\varepsilon_t < 0$ lead to the choice of a lower rate of inflation to prevent the economy from overheating because of the upward deviation of the output from its natural level. The policy has stabilising impact on the economy.

**Grading scheme**: 50% of points for unclear explanations.

**Problem 3**

a) An analyst has estimated four cross section ordinary least squares regressions trying to model the link between share prices $(P)$ and last reported earnings per share $(EPS)$ for a sample of emerging markets equities. The summary of estimation results are provided in the table below.

| | Model | Number of observations | Coefficients | | p-values for tests | | R-squared |
|---|---|---|---|---|---|---|---|
| | | | $\alpha$ | $\beta$ | H-0: $\alpha = 0$ | H-0: $\beta = 0$ | |
| I | $P_i = \alpha + \beta EPS_i + \varepsilon_i$ | 821 | 16.8 | 6.75 | 0.00 | 0.00 | 0.66 |
| II | $P_i = \alpha + \beta\log(EPS_i) + \varepsilon_i$ | 745 | 83.2 | 38.1 | 0.00 | 0.00 | 0.24 |
| III | $\log(P_i) = \alpha + \beta EPS_i + \varepsilon_i$ | 821 | 1.32 | 0.03 | 0.00 | 0.00 | 0.08 |
| IV | $\log(P_i) = \alpha + \beta\log(EPS_i) + \varepsilon_i$ | 745 | 2.72 | 0.92 | 0.00 | 0.00 | 0.85 |

i) Noting that the share price and EPS data is in US dollar terms, provide an economic interpretation of the $\beta$coefficient estimates for each model. [4p]

ii) Which model(s) would you prefer and why? [3p]

b) What are the covariance stationarity conditions of a stochastic process? [6p]

c) Prove that the unconditional variance $\mathrm{V}(y_t)$ is strictly larger than a conditional variance $\mathrm{V}_{t-1}(y_t)$ for the following covariance stationary process: $y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid\ WN(0, \sigma^2)$? [7p]

**Answers for problem 3**

a) An analyst has estimated four cross section ordinary least squares regressions trying to model the link between share prices $(P)$ and last reported earnings per share $(EPS)$ for a sample of emerging markets equities. The summary of estimation results are provided in the table below.

| | Model | Number of observations | Coefficients $\alpha$ | Coefficients $\beta$ | p-values for tests H-0: $\alpha = 0$ | p-values for tests H-0: $\beta = 0$ | R-squared |
|---|---|---|---|---|---|---|---|
| I | $P_i = \alpha + \beta EPS_i + \varepsilon_i$ | 821 | 16.8 | 6.75 | 0.00 | 0.00 | 0.66 |
| II | $P_i = \alpha + \beta \log(EPS_i) + \varepsilon_i$ | 745 | 83.2 | 38.1 | 0.00 | 0.00 | 0.24 |
| III | $\log(P_i) = \alpha + \beta EPS_i + \varepsilon_i$ | 821 | 1.32 | 0.03 | 0.00 | 0.00 | 0.08 |
| IV | $\log(P_i) = \alpha + \beta \log(EPS_i) + \varepsilon_i$ | 745 | 2.72 | 0.92 | 0.00 | 0.00 | 0.85 |

i) Noting that the share price and EPS data is in US dollar terms, provide an economic interpretation of the $\beta$coefficient estimates for each model. [4p]

**Solution**:

| | Model | Interpretation |
|---|---|---|
| I | $P_i = \alpha + \beta EPS_i + \varepsilon_i$ | \$1 increase in EPS is associated with \$6.8 increase in the share price on average |
| II | $P_i = \alpha + \beta \log(EPS_i) + \varepsilon_i$ | 1% increase in EPS is associated with \$0.381 increase in the share price on average |
| III | $\log(P_i) = \alpha + \beta EPS_i + \varepsilon_i$ | \$1 increase in EPS is associated with 3% increase in the share price on average |
| IV | $\log(P_i) = \alpha + \beta \log(EPS_i) + \varepsilon_i$ | 1% increase in EPS is associated with 0.9% increase in the share price on average |

**Grading scheme**: 25% of points for correct interpretation for each model. Maximum 75% of total points if the coefficients in models II and III are not adjusted due to percentages.

ii) Which model(s) would you prefer and why? [3p]

**Solution**: As the number of observations differs between models, not all of them are comparable. Due to higher R-squared, models I is preferred to III and model IV is preferred to model II. This result is due to the fact that models II and III are fundamentally misspecified as they link dollar values with percentage changes. No conclusion can be made regarding models I and IV as higher R-squared of the latter may stem from smaller sample size.

**Grading scheme**: 1/3 of points for the noticing that models II and III cannot be preferred. 2/3 of points maximum if the difference in the estimation sample between models is not taken into account.

b) What are the covariance stationarity conditions of a stochastic process? [6p]

**Solution**: Unconditional mean and unconditional variance must be constant over time and the latter must be finite: $\mathrm{E}(y_t) = \mu$ and $\mathrm{V}(y_t) = \sigma^2 < \infty$ for all $t$. Also autocovariance must not depend on $t$: $\mathrm{Cov}(y_t, y_{t-s}) = \gamma_s$ for all $t$ and all $s$.

**Grading scheme**: 1/3 of points for each fully correct restriction on unconditional moments. 1/6 of points marks for each partially correct restriction on unconditional moments.

c) Prove that the unconditional variance $\mathrm{V}(y_t)$ is strictly larger than a conditional variance $\mathrm{V}_{t-1}(y_t)$ for the following covariance stationary process: $y_t = \phi_0 + \phi_1 y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid\ WN(0, \sigma^2)$? [7p]

**Solution**: Due to covariance stationarity, $\mathrm{V}(y_t) = \mathrm{V}(y_{t-1})$ and the unconditional variance can also be derived directly from $\mathrm{V}(y_t) = \mathrm{V}(\phi_0 + \phi_1 y_{t-1} + \varepsilon_t)$:

$$\begin{aligned}\mathrm{V}(y_t) = \mathrm{V}(\phi_0) + \mathrm{V}(\phi_1 y_{t-1}) + \mathrm{V}(\varepsilon_t) + 2\mathrm{Cov}(\phi_0, y_{t-1}) + 2\mathrm{Cov}(\phi_0, \varepsilon_t) \\ + 2\mathrm{Cov}(\phi_1 y_{t-1}, \varepsilon_t)\end{aligned}$$

$\mathrm{V}(\phi_0) = 0$ and all covariance terms in the expression above are zero because $\phi_0$ is a constant and white noise $\varepsilon_t$ is unrelated to $y_{t-1}$.

$$\mathrm{V}(y_t) = \phi_1^2 \mathrm{V}(y_{t-1}) + \sigma^2$$

$$\mathrm{V}(y_t) = \frac{\sigma^2}{1 - \phi_1^2}$$

The conditional variance is $\mathrm{V}_{t-1}(y_t) = \mathrm{V}_{t-1}(\phi_0 + \phi_1 y_{t-1} + \varepsilon_t) = \sigma^2$ as $\phi_0$ is a constant as $y_{t-1}$ is known at $(t-1)$, and hence is also treated as a constant.

As unconditional variance cannot be negative (and also because covariance stationarity for ARMA processes requires that the characteristic roots of the associated characteristic polynomial are less than one in modulus), $\phi_1^2 < 1$, which proves that $\mathrm{V}(y_t) = \frac{\sigma^2}{1-\phi_1^2} > \sigma^2 = \mathrm{V}_{t-1}(y_t)$

**Grading scheme**: 30% of points for conditional variance derivation. 50% of points for unconditional variance derivation. 20% of points for the correct conclusion.

**Answers for problem 4**

a. We calculate the NPV of the incremental cash-flows generated by the hire of the actress.

Without the actress, the expected cash-flow is (1/3)*80+(2/3)*20. With the actress, the

expected cash-flow is (2/3)*80+(1/3)*20. Thus the NPV of the project is

-5+(1/3)* (80-20)>0

The producer's payoff is 80-60=20 in the case of success and 0 otherwise (limited liability). Since the probability of success increases from 1/3 to 2/3, the producer's gain is

-5+(1/3)*20>0

Thus the producer undertakes the project and hires the confirmed actress.

**Marking scheme: 3 points for the calculation of the NPV (1 point only if students do not consider incremental cash flows), 2 points for the participation constraint of the producer.**

b.

If the probability of success due to the participation of the actress increases to ½ instead of 2/3, the NPV of the project is

-5+(1/6) (80-20)=5>0

Thus, even with a lower increment in the probability of success, it is still worth hiring the rising star.

The producer has an incentive to do it iff

-5-(1/3)*20+(1/2)*20>0

This equation is not satisfied, thus the producer does not hire the rising star even though it is a positive NPV decision. The firm faces a typical debt overhang problems where positive NPV projects are passed out because shareholders cannot capture a large enough share of the profits due to the large amount of debt.

**Marking: 1 point for NPV, 1 for participation constraint, 1 point for the explanation about the debt overhang problem.**

c.

For a given face value of debt D, the cash-flow to the producer is max (80-D,0) in case of success, and max (20-D,0) otherwise.

**(1 point for this analysis)**

Therefore, the producer hires the actress iff

$-5-[\ (1/3)*\max(0,80-D)+(2/3)*\max(0,20-D)\ ]+[(1/2)*\max(0,80-D)\ +(1/2)*\max(0,20-D)\ ]\geq 0$

i.e.

$-5+(1/6)*[\ \max(0,80-D)-\max(0,20-D)] \geq\ 0$

**(1 point for this equation or the previous one)**

We need to distinguish two cases:

- If D>20 (risky debt), the equation becomes

$-5+(1/6)*(80-D) \geq 0,$

which holds iff $D \leq 50$

- If $D \leq 20$, then the equation becomes

$-5+(1/6)*(80-D)-(1/6)*\ (20-D)\backslash \geq 0$ ,

which always holds true.

**(2 points for this discussion as a function of the face value)**

In this case, debt is so low that the producer always finances the hire. Thus the maximal amount of debt that the producer can sustain is 50. If banks reduce the face value of debt to that level, they get on average (1/2)*50+(1/2)* 20=35, while if they don't, they get on average (1/3)*60+(2/3)*20=33.3.

Hence banks have an incentive to renegotiate the face value of debt.

**(2 points for the analysis of banks' incentives to renegotiate)**

d.

Suppose that the producer promises a share $\alpha$ of the profits in exchange of \$5m from outside shareholders. To finance the movie, outside shareholders must break even on average, i.e.

$$\alpha(1/2)*(80-60)+\alpha(1/2)*0 \geq 5$$

which is equivalent to

$$\alpha \geq 1/2$$

**(1 point for financiers' break-even condition)**

The producer participates iff $(1/2)*(1-\alpha)\,20 \geq (1/3)*20$.

Since $\alpha$ must be greater than 50%, the left-hand side is lower than 5. Hence the producer does not want to participate.

**(2 points for producer's participation constraint)**

To induce outside shareholders to invest in the project, one must grant them a large share of the profits. The debt overhang problem is that the benefits of new projects go first to the existing creditors. Raising new equity does not solve the problem because the new investors require a fair return relative to the investment. The only way to solve the problem is to reduce the amount of debt that distorts the incentives of current shareholders to pass out positive NPV projects. In the previous question, we saw that debt forgiveness can solve the problem.

**(3 points for the entire discussion, of which 2 points for the explanation why the equity solution does not work (just passing the same problem on to new shareholders, who also require a fair return), and 1 point if it is mentioned that debt forgiveness works)**

**Answer for question 5**

**Grading Scheme:** For all sub-tasks, approximately 60% of points for the derivations and 40% for the logic, at the discretion of the grader.

a) To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b) If $r_f$ = 6% and $r_M$ = 14%, then (using the notation alpha for the abnormal return):

   $\alpha$ 1 = 18.5 – [6 + 1.5(14 – 6)] = 18.5 – 18 = 0.5%
   $\alpha$ 2 = 15.5 – [6 + 1(14 – 6)] =15.5 – 14 = 1.5%
   Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c) When there is a 5% stock dividend, the option contract becomes one to buy 600x1.05=630 shares with an exercise price 30/1.05=28.57. The terms of an option contract are not normally adjusted for cash dividends. The question is about the contractual terms; not about the market price. They change automatically with a stock dividend but not with a cash dividend. So this question examines some knowledge about the institutional setup of option markets, which is as important as knowledge of pricing models. The timing and frequency of the dividends does not matter. The question asks what happens to the contractual terms “when there is a 5% stock dividend paid”. The word “when” means “in the moment”, so this is clearly a one-time event.

d) $E(r_M)$ = 12%, $r_f$ = 4% and $\beta$= 0.5. Therefore, the expected rate of return is: 4% + 0.5(12% – 4%) = 8%. If the stock is fairly priced, then E(r) = 8%.

e) If $r_M$ falls short of your expectation by 2% (that is, 10% – 12%) then you would expect the return for XYZ to fall short of your original expectation by: $\beta$ x 2% = 1%.
   Therefore, you would forecast a “revised” expectation for XYZ of: 8% – 1% = 7%.

f) Given a market return of 10%, you would forecast a return for XYZ of 7%. The actual return is 10%. Therefore, the surprise due to firm-specific factors is 10% – 7% = 3% which we attribute to the settlement. Because the firm is initially worth $100 million, the surprise amount of the settlement is 3% of $100 million, or $3 million, implying that the prior expectation for the settlement was only $2 million.

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