

Направление: «Финансовая экономика»

Профиль: «Финансовая экономика / Financial Economics»

КОД - 130

Examination Guidelines

- The exam consists of 5 tasks. Solve all tasks. Time **180** minutes.
- The exam is graded on a 100-point scale. Numbers in brackets indicate the points awarded for each problem.
- Write your answers in the booklet provided to you by the examiners.
- You can solve the tasks in any order but you must label each task and sub-task clearly and sufficiently. Use a separate page for each task. You are not allowed to detach sheets from the booklet.
- Answer all tasks **in English**. Graders will ignore any Russian text.
- You may use the last page of your booklet as scrap paper.
- Crossed out writing will not be considered by the grader.
- Use legible hand writing. The grader will ignore any illegible parts of your paper.

Examination Rules

- You are required to follow all instructions given by the examiners.
- Talking is not allowed under any circumstances.
- During the exam you are allowed to have on your desk two pens (black or blue), a drink and a non-programmable calculator. You are not allowed to bring any written or printed materials, mobile phones or other electronic devices into the examination room.
- Detection of any electronic communication device on you will constitute cheating even if the device is off.
- The proctors of the exam are not authorized to answer any questions.
- Exam participants are not allowed to leave the examination room until ready to turn in their work.

Task 1 [20p]

Consider two countries, X and Y. Each country produces two goods, labeled 1 and 2, using the only factor of production – labor. The amount of good i that one worker can produce in country j is $a_{ji} > 0$. We assume that $a_{X1}/a_{X2} > a_{Y1}/a_{Y2}$. The production is perfectly competitive. The labor is freely mobile between production of the two goods within the country, but cannot move between countries.

- (a) Which country has a comparative advantage in producing good 1? [1p]

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Suppose the size of the labor force in country j is L_j . Suppose also that the price of good 2 is equal to one, while the price of good 1 is equal to some p .

- (b) Calculate the global relative supply, that is, the ratio $RS(p) = Q_1^S(p)/Q_2^S(p)$, where $Q_i^S(p)$ is the quantity of good i produced globally in equilibrium, as a function of p . [2p]

Suppose the consumer preferences in both countries are given by the utility function

1\2	L	R
U	u_1, v_1	u_2, v_2
D	u_3, v_3	u_4, v_4

$$u = (q_1^{-1} + q_2^{-1})^{-1},$$

where q_i is the quantity of good i consumed.

- (c) Calculate the global relative demand, that is, the ratio $RD(p) = Q_1^D(p)/Q_2^D(p)$, where $Q_i^D(p)$ is the quantity of good i consumed globally in equilibrium, as a function of p . [3p]
- (d) Find the equilibrium price p . Find u_X , the utility of a representative consumer in country X in equilibrium. [4p]
- (e) If country X becomes more productive in good 1, so that a_{X1} increases, what happens to the utility u_X ? Identify the range of a_{X1} , as function of all other model parameters, such that $du_X/da_{X1} < 0$. [4p]
- (f) Explain intuitively why a productivity increase may cause consumer welfare in that country to decline. [3p]
- (g) If consumer preferences changed so that the utility was

$$u = (q_1^{1/2} + q_2^{1/2})^2,$$

would it expand or contract the range of a_{X1} such that $du_X/da_{X1} < 0$? (no proof required, provide the intuition only). [3p]

Task 2 [20p]

- (a) Consider the following static game with two players and two actions for each player, where the payoffs of player 1 are denote by u and the payoffs of player 2 are denoted by v . Prove that if no action is weakly dominated, then there exists a mixed equilibrium of the game that assigns positive probability to all actions. [9p]
- (b) Consider a static game with two players, 1 and 2, and a finite number of actions for each player. Consider a dominated action a_1 of player 1 and a mixed action α_1 that dominates it (a mixed action of player 1 is a probability distribution over her own actions). Suppose that player 1 has a conjecture μ^1 about the action a_2 chosen by player 2 (a conjecture of player 1 is a probability distribution over the actions of player 2). Show that, under μ^1 , player 1 has a higher expected payoff by playing α_1 than by playing a_1 . [4p]
- (c) Show that the expected payoff of α_1 under μ^1 can be rewritten as a weighted sum of the expected payoffs under μ^1 of the actions in the support of α_1 (that is, the actions that take positive probability under α_1). [4p]
- (d) Use (b) and (c) to argue that there must exist a *pure* (not mixed) action a_1' different from a_1 that, under μ^1 , gives to player 1 a higher expected payoff than a_1 . [3p]

Task 3 [20p]

- (a) An analyst has estimated two time series regression models to model the historical link between dividends and earnings for the S&P 500 Index. One model (Model A) is

$$DPS_t = \alpha + \beta EPS_t + \varepsilon_t$$

and the other model (Model B) is

$$DPS_t = \alpha + \beta_1 EPS_t + \beta_2 DPS_{t-1} + \beta_3 EPS_{t-1} + \varepsilon_t$$

where EPS_t and DPS_t are the yearly earnings per share and dividends per share respectively for the S&P 500 Index, and ε_t are i.i.d. normal random variables. The estimation period is 1991-2016 (26 observations) for both models. The estimation results are provided in the two tables below.

Model	R-squared	Coefficient Statistics		
		α	β	
A	0.84	Coefficient	3.3	0.3
		Standard Error	1.9	0.028
		t-stat (H_0 : Coeff=0)	1.7	11.1
		p-value (H_0 : Coeff=0)	0.09	0.00

Model	R-squared	Coefficient Statistics				
		α	β_1	β_2	β_3	
B	0.98	Coefficient	-0.5	0.04	0.77	0.07
		Standard Error	0.8	0.03	0.08	0.04
		t-stat (H_0 : Coeff=0)	-0.7	1.46	9.36	1.67
		p-value (H_0 : Coeff=0)	0.5	0.16	0.00	0.11

Explain in detail how the t-statistics and p-values above have been calculated and what can be concluded from them. [3p]

- (b) Based on the data in the table in question (a), calculate a static that would allow to assess whether Model B significantly outperforms Model A and explain how you would perform the appropriate test. [3p]
- (c) Derive the variance, autocovariances and autocorrelations for the following model: $y_t = \alpha + \varepsilon_t + \mu\varepsilon_{t-1}$, where ε_t is white noise and μ is less than one by absolute value. [6p]
- (d) Consider a model $y_i = \alpha + u_i$, $i = 1, 2, \dots, N$, where $E(u_i) = 0$, $E(u_i^2) = \sigma^2$ and $E(u_i u_j) = 0$ for all $i \neq j$. Derive the least squares estimator of α . [4p]
- (e) Prove that the estimator that you derived in question (d) above is unbiased and consistent. [4p]

Task 4 [20p]

The time is the second half of the 19th century. The place is a small town in Russia. In our town there live 2 very different brothers, Dimitry Fjodorovitsch and Alexei Fjodorovitsch. Dimitry Fjodorovitsch is a soldier while Alexei Fjodorovitsch has chosen to live in a monastery, devoting his life to the spiritual wellbeing of those around him.

Their father Fjodor Pavlovitsch, a wealthy man, wants to give his sons part of their inheritance before his death. Since he has enjoyed a very unhealthy life-style, he knows he is not going to see the following year. He has 320 000 rubles (a vast sum of money at the time) to bequeath to them. He decides to give each son 60 000 rubles now and 100 000 rubles after his death.

The nominal interest rate between the current year t_0 and the next year t_1 is $i=5\%$ (Fjodor Pavlovitsch does not trust banks and thus would neither borrow nor put any money into the bank himself, but his sons have no such compunctions.). Interest rate for borrowing and lending is the same; current and next year's consumption are denoted with c_0 and c_1 .

- (a) Provide the intertemporal budget constraint. On a graph with c_0 on the horizontal axis and c_1 on the vertical axis, draw the intertemporal budget constraint each son will face and clearly indicate the intercepts on the two axes and the slope of the budget constraint. [2p]
- (b) Give the economic interpretation of the slope of the budget constraint. Reason in terms of opportunity cost and trade-offs. [3p]
- (c) Dimitry Fjodorovitsch, being a passionate soldier, cares only about how to impress his love interest Agrafena Alexandrovna immediately to prevent her from leaving him. His preferences can be described as:

$$U(c_0, c_1) = \sqrt{\frac{a + \sqrt{\ln(c_0)}}{b}}$$

Derive his optimal consumption bundle $c^* = (c_0^*, c_1^*)$. In the graph in (a), represent his preferences using a map of indifference curves (at least 2 indifference curves) and show $c^* = (c_0^*, c_1^*)$. Is he a borrower or a lender? [4p]

- (d) Meanwhile, Alexei Fjodorovitsch is only concerned with having enough money to survive and spend the rest on charity. He finds that it is no good to help just once if he is not able to lend assistance in the future. However, he knows next year, his dear friend Starez Sossima will be in need of some financial support as his health is deteriorating. Thus, Alexei Fjodorovitsch's preferences can be described as:

$$U(c_0, c_1) = \min(c_0, (1 - \alpha)c_1)$$

We know that $0 < \alpha < 1$ as it captures how much he cares about Starez Sossima's wellbeing. Calculate Alexei Fjodorovitsch's optimal consumption bundle $c^{**} = (c_0^{**}, c_1^{**})$. In a new graph, draw his preferences using a map of indifference curves (2 indifference curves) and show $c^{**} = (c_0^{**}, c_1^{**})$. Calculate the threshold of α before which he is a borrower and after which he is a lender and explain. [5p]

- (e) Compare the optimal consumption bundles c^* and c^{**} and discuss the differences. [3p]

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- (f) Without any calculation, explain what happens to Alexei Fjodorovitsch's optimal consumption bundle $c^{**} = (c_0^{**}, c_1^{**})$ if the nominal interest rate increases to $i=10\%$. Make explicit references to substitution and income effect. [3p]

Task 5 [20p]

Consider a firm which is active for one period, between $t = 0$ and $t = 1$. The existing business of the firm generates cash-flows at $t = 1$ of either 200 or 100, with equal probability. The firm has debt outstanding with a face value of 150, which is due at $t = 1$. Of course, bondholders are senior claimants to equity holders in case of full or partial default of the firm. The firm's manager takes all investment decisions in the interest of equity holders. All agents in the economy are risk-neutral. The discount rate is zero and there are no taxes.

- (a) What is the value of the firm? What the value of the firm's debt? The value of equity? [3p]

Assume that the firm has two new investment projects, A and B. Each individual project requires an additional financing of 30. If the firm invests in project A, the cash-flows next period will increase by 40 for sure. In other words, if the firm invests in project A, cash-flows will be either 240 or 140 with equal probability. If the firm invests in project B, cash-flows increase by 50 in the high state and decrease by 50 in the low state. In other words, if the firm invests in B, cash-flows will be either 250 or 50 with equal probability.

- (b) Assume that there are covenants prohibiting the issue of additional debt. Which project would the manager choose if she had the money? Will shareholders be willing to put the additional 30 for the investment? [5p]
- (c) The manager asks the debt holders to waive the covenant, so that the firm can issue (new) senior debt. Assume that the manager can only invest in project A, and the debt holders (old and new ones) know this. Would the firm be able to raise new senior debt to finance the project? Will the existing debt holders agree to waive the seniority covenant? Assume that the new senior debt will be paid off before the existing debt in the case of default. [6p]
- (d) Assume again that the manager asks the debt holders to waive the covenant, so that the company could issue senior debt, but that debt holders now believe that there is a 50% chance that the manager has access to both projects A and B. Would the firm be able to raise new senior debt to finance the project if existing debt holders waived the covenant? Will the existing debt holders be willing to waive the seniority covenant? [6p]

End of questions - Good luck!