

Answers for ICEF Olympiad 2018

General grading guidelines

These are 5 original tasks created by an international academic committee especially for the purpose of this Olympiad. Each task has been created by a specialist in the area, edited by the academic director of the Olympiad and then blindly reviewed by another specialist independently. These are open ended tasks. The answers will be marked blindly at the discretion of the grading team in a manner uniform for all candidates.

Guidelines

1. Full credit should be reserved for answers that demonstrate mastering the material and the explanation is sufficient and flawless at the discretion of the grader.
2. Partial credit may be given to insufficient answers towards the correct direction at the discretion of the grader.
3. Minimal credit may be given to answers not towards the correct direction, which however indicate that the candidate has some limited knowledge on the subject at the discretion of the grader.
4. Zero credit should be given to answers that are wrong, irrelevant, make no sense, or have random correct elements, at the discretion of the grader.

Any indication of failure to master the material should result to a reduction of credit. For example, if a candidate answers the question perfectly and then attaches an additional part which is irrelevant or wrong, the grader may remove points because the redundancy indicates failure in understanding.

Appeals

Candidates have the right to appeal to the decisions made by the graders according to the HSE Olympiad regulations.

The academic director of the Olympiad will decide if an appeal will be forwarded to the members of the committee for re-grading or it will be terminated as lacking merit. Appeals will be forwarded for re-grading only when one of the following occurs:

1. The candidate sufficiently justifies that the grader has made a specific mistake grading a specific part of the answer.
2. The candidate sufficiently proves that there is a misspecification in some part of the question that allows the candidate's answer to be interpreted as a partially or fully correct response.
3. The candidate sufficiently proves that there is a mistake in a specific part of the question such that there is no correct response.

4. The candidate sufficiently demonstrates that the given answer is also (partially) correct, even though it is not included in this answer key.

Appeals that are insufficiently justified; or do not fall under one of the above 4 cases; or are not in English; or are unclear; or are just asking for higher grades, will be terminated by the academic director of the Olympiad with the indication: “The appeal has no merit (# reason #)”.

If the appeal is judged as reasonable by the academic director, a committee member will be asked to re-grade the question. In this case the candidate is possible to receive a higher or a lower or the same grade in comparison to that received in the first degree.

Answer for task 1

- a) Country X is relatively more productive in good 1.
- b) Workers in country j compare incomes pa_{j1} (in production of good 1) against a_{j2} (in production of good 2), choose whichever is higher.
- If $p < a_{x2}/a_{x1}$, workers in both countries produce in industry 2, thus $RS(p) = Q_1^S(p) = 0$.
 - If $p = a_{x2}/a_{x1}$, workers in Y produce good 2 while workers in X are indifferent between goods 1 and 2. Depending on how workers in X are allocated, $RS(p)$ ranges from 0 to $L_X a_{x1}/(L_Y a_{y2})$.
 - If $a_{x2}/a_{x1} < p < a_{y2}/a_{y1}$, workers in X produce good 1 while workers in Y produce good 2. $RS(p) = L_X a_{x1}/(L_Y a_{y2})$.
 - If $p = a_{y2}/a_{y1}$, workers in X produce good 1 while workers in Y are indifferent between 1 and 2. Depending on how workers in Y are allocated, $RS(p)$ ranges from $L_X a_{x1}/(L_Y a_{y2})$ to infinity.
 - If $p > a_{y2}/a_{y1}$, workers in both countries produce in industry 1, thus $Q_2^S(p) = 0$ and $RS(p)$ is infinite.
- c) A property of optimal consumer choice is that the ratio of marginal utilities, $(du/dq_1)/(du/dq_2)$, is equal to the ratio of prices, p . We have that $du/dq_i = (q_1^{-1} + q_2^{-1})^{-2} q_i^{-2}$, thus $(du/dq_1)/(du/dq_2) = (q_1/q_2)^{-2} = p$. Because all consumers choose the same ratio q_1/q_2 , it is equal to the global relative demand $Q_1^D(p)/Q_2^D(p)$, thus $RD(p) = p^{-1/2}$.
- d) The equilibrium price is found from the equation $RS(p) = RD(p)$. Three cases can be distinguished.
- If $L_X/L_Y \geq a_{y2}/(a_{x1}^{1/2} a_{x2}^{1/2})$, we have that $p = a_{x2}/a_{x1}$.
 - If $a_{y2}/(a_{x1}^{1/2} a_{x2}^{1/2}) \geq L_X/L_Y \geq (a_{y1}^{1/2} a_{y2}^{1/2})/a_{x1}$, we have that $RD(p) = L_X a_{x1}/(L_Y a_{y2})$ and $p = (L_Y a_{y2})^2 / (L_X a_{x1})^2$.
 - If $(a_{y1}^{1/2} a_{y2}^{1/2})/a_{x1} \geq L_X/L_Y$, we have that $p = a_{y2}/a_{y1}$.

In all these cases, a positive quantity of good 1 is produced in country X and therefore the GDP of country X is equal to $pa_{x1}L_X$ (maximum quantity of good 1 produced, multiplied by its price). The quantities consumed in country X, Q_{x1}^D and Q_{x2}^D , are found from the relative demand equation $Q_{x1}^D/Q_{x2}^D = p^{-1/2}$ and the budget constraint $pQ_{x1}^D + Q_{x2}^D = pa_{x1}L_X$, with the following solution: $Q_{x1}^D = p^{1/2}a_{x1}L_X/(1+p^{1/2})$ and $Q_{x2}^D = pa_{x1}L_X/(1+p^{1/2})$. By substituting these into the definition of utility, we obtain $u_X = pa_{x1}L_X / (1+p^{1/2})^2$.

- e) du_X/da_{x1} has the same sign as $d \log(u_X)/da_{x1} = 1/a_{x1} + 1/(p(1+p^{1/2})) (dp/da_{x1})$. In the intermediate case (ii) of the previous question, we have $dp/da_{x1} = -2p/a_{x1}$ and thus $d \log(u_X)/da_{x1} = 1/a_{x1} (1-2/(1+p^{1/2}))$, which is negative in case $(L_Y a_{y2})/(L_X a_{x1}) = p^{1/2} < 1$. To summarize, the following conditions should hold for decreasing utility:
- The bounds for case (ii) of question (d): $(a_{y1}^{1/2} a_{y2}^{1/2}) L_Y/L_X \leq a_{x1} \leq (L_Y/L_X a_{y2})^2 / a_{x2}$
 - Newly established constraint $a_{x1} > a_{y2} L_Y/L_X$
- f) Increased productivity has two effects: more output (positive) but a lower export price of that output (negative). With low substitution elasticity between the two goods, the

demand has low price elasticity, thus increased supply due to higher productivity drops the export price significantly, causing a decrease in welfare.

- g) The alternative utility implies higher substitution elasticity, thus higher price elasticity of demand, thus an increase in output of good 1 will cause a smaller change in price (consumers will switch from good 2 to good 1). Therefore, the negative effect through changing price will be weaker, making it less likely that $du_x/da_{x1} < 0$. In fact, it can be proven that $du_x/da_{x1} > 0$ for all values of a_{x1} under the alternative utility function.

Answer for task 2

1\2	L	R
U	u_1, v_1	u_2, v_2
D	u_3, v_3	u_4, v_4

1. Consider the following payoff matrix, where the payoffs of player 1 are denoted by u and the payoffs of player 2 by v .

$1 \backslash 2$	L	R
U	u_1, v_1	u_2, v_2
D	u_3, v_3	u_4, v_4

For player 1 to randomize between U and D in equilibrium, she must be indifferent between the two against the equilibrium conjecture. Let β denote the probability of L in equilibrium. Then, we must have

$$\beta u_1 + (1 - \beta)u_2 = \beta u_3 + (1 - \beta)u_4.$$

Rewrite it as

$$\beta(u_1 - u_3) = (1 - \beta)(u_4 - u_2).$$

Since no action is weakly dominated, we have: (i) if $u_1 - u_3 > 0$, then $u_4 - u_2 > 0$; (ii) if $u_1 - u_3 < 0$, then $u_4 - u_2 < 0$; (iii) if $u_1 - u_3 = 0$, then $u_4 - u_2 = 0$. In all three cases, the equation has a solution $\beta \in (0, 1)$ (in case (iii), any β works).

Repeating the same reasoning for player 2, we find a probability $\alpha \in (0, 1)$ of U that leaves player 2 indifferent between L and R .

Recall that when a player is indifferent between two actions, she is also indifferent among all the mixes of the two. So, the mixed action profile where player 1 plays U with probability α and player 2 plays L with probability β is a mixed equilibrium.

2. By definition of dominance, for every $a_{-i} \in A_{-i}$, we have

$$u_i(a_i, a_{-i}) < \sum_{a'_i \in \text{Supp}\alpha_i} \alpha_i(a'_i) u_i(a'_i, a_{-i}).$$

But then,

$$\sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) u_i(a_i, a_{-i}) < \sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) \left(\sum_{a'_i \in \text{Supp}\alpha_i} \alpha_i(a'_i) u_i(a'_i, a_{-i}) \right). \quad (1)$$

3. Using the basic distributive property of summations, the right hand side of equation 1 can be rewritten as follows:

$$\sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) \left(\sum_{a'_i \in \text{Supp}\alpha_i} \alpha_i(a'_i) u_i(a'_i, a_{-i}) \right) = \sum_{a'_i \in \text{Supp}\alpha_i} \alpha_i(a'_i) \left(\sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) u_i(a'_i, a_{-i}) \right).$$

4. Rewriting the right hand side of equation 1 as above, we obtain

$$\sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) u_i(a_i, a_{-i}) < \sum_{a'_i \in \text{Supp}\alpha_i} \alpha_i(a'_i) \left(\sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) u_i(a'_i, a_{-i}) \right),$$

which implies that for at least one $a'_i \in A_i$,

$$\sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) u_i(a_i, a_{-i}) < \sum_{a_{-i} \in A_{-i}} \mu^i(a_{-i}) u_i(a'_i, a_{-i}).$$

Answer for task 3

a) Each t-statistic has been calculated as $\frac{\text{Coefficient}}{\text{Standard Error}}$. t-statistics are then benchmarked against a student's t-distribution with the number of degrees of freedom equal to the number of observations less the number of estimated parameters, i.e. 24 degrees of freedom for Model A and 22 degrees of freedom for Model B. P-values are equal to the following probability: $2 * P(t_{df} < -|t^*|)$, where t^* is the t-statistic shown in the table, t_{df} is a random variable with the student's t-distribution with df degrees of freedom and it is multiplied by two to conduct a two-sided test.

The t-statistics and the p-values above tell that in Model A the intercept is statistically significant at 10% level, while the EPS_t coefficient is significant even at 1% level. In Model B, DPS_{t-1} is significant even at 1% level while all other coefficients appear insignificant at 10% level.

Grading scheme: 33% of points for the formula for the t-statistics. 33% of points for the explanation of the p-value calculations, including the number of degrees of freedom. 33% of points for the correct conclusions regarding both models' coefficients.

b) The following F-statistic can be calculated to assess this: $\frac{R_{UR}^2 - R_R^2}{\frac{q}{1 - R_{UR}^2}} \sim F_{q, n-k}$, where

$R_{UR}^2 = R_B^2$ is the R-squared in the unrestricted model (Model B), $R_R^2 = R_A^2$ is the R-squared in the restricted model (Model A), $q = 2$ is the number of restrictions ($\beta_2 = 0$ and $\beta_3 = 0$) and $n - k = 22$ is the number of observations (26) less the number of estimated parameters in the unrestricted model (4). q and $n - k$ are the degrees of freedom of the F-distribution that the F-statistic is benchmarked against.

$F^* = 77$ implies that we cannot accept the null hypothesis $\beta_2 = 0$ and $\beta_3 = 0$ at any reasonable significance level (even $F_{2,22}^{1\%} = 5.72$), but this conclusion is not required as F-distribution tables are not provided.

Grading scheme: 33% of points for the formula for the F-statistic. 33% of points for the explanation of the F-test, including the number of degrees of freedom. 33% of points for the F-statistic calculations. No conclusions regarding the outcome of the test are required as F-distribution tables are not provided. Up to 67% of points can be awarded if the adjusted R-squared is calculated for each model instead of the F-statistic. Only 33% points if there is no interpretation of the adjusted R-squared, i.e. that it takes into account the increase in the number of explanatory variables.

c)

$$\begin{aligned} \gamma(0) &= V(y_t) = E(y_t - E(y_t))^2 = E(\alpha + \varepsilon_t + \mu\varepsilon_{t-1} - \alpha)^2 = \\ &= E(\varepsilon_t + \mu\varepsilon_{t-1})^2 = E(\varepsilon_t^2) + \mu^2 E(\varepsilon_{t-1}^2) + 2\mu E(\varepsilon_t \varepsilon_{t-1}) = (1 + \mu^2)\sigma^2 \end{aligned}$$

due to the fact that ε_t is white noise, and hence $E(\varepsilon_t^2) = E(\varepsilon_{t-1}^2) = \sigma^2$ and $E(\varepsilon_t \varepsilon_{t-1}) = 0$.

$$\gamma(1) = \text{Cov}(y_t, y_{t-1}) = \text{Cov}(\alpha + \varepsilon_t + \mu\varepsilon_{t-1}, \alpha + \varepsilon_{t-1} + \mu\varepsilon_{t-2}) = \mu\sigma^2$$

as all covariance terms are zero except for $\text{Cov}(\mu\varepsilon_{t-1}, \varepsilon_{t-1}) = \mu V(\varepsilon_{t-1}) = \mu\sigma^2$.

$\gamma(2) = \gamma(3) = \dots = 0$ as all covariances are zero, for example $\gamma(2) = \text{Cov}(y_t, y_{t-2}) = \text{Cov}(\alpha + \varepsilon_t + \mu\varepsilon_{t-1}, \alpha + \varepsilon_{t-2} + \mu\varepsilon_{t-3}) = 0$.

$$\text{Autocorrelation } r(1) = \frac{\gamma(1)}{\gamma(0)} = \frac{\mu\sigma^2}{(1+\mu^2)\sigma^2} = \frac{\mu}{(1+\mu^2)}.$$

Higher-order autocorrelations $r(2)$, $r(3)$, etc, are zero due to zero higher-order autocovariances $\gamma(2)$, $\gamma(3)$, etc.

Grading scheme: 33% of points for the variance derivation. 50% of points for the autocovariances (first-order and higher-order). The rest for autocorrelations. Maximum 67% of points if only first-order autocovariances and autocorrelations are considered, i.e. if the higher-order ones are ignored or incorrect.

d) $\hat{\alpha}$ is the solution to the following minimisation problem:

$$\begin{aligned}\sum_{i=1}^N \hat{u}_i^2 &= \sum_{i=1}^N (y_i - a)^2 = \sum_{i=1}^N (y_i^2 + a^2 - 2y_i a) = \\ &= \sum_{i=1}^N y_i^2 + Na^2 - 2a \sum_{i=1}^N y_i \rightarrow \min_a\end{aligned}$$

Differentiating with respect to a , one can find the least squares estimator:

$$\begin{aligned}\frac{d}{da}(\hat{\alpha}) &= 2N\hat{\alpha} - 2 \sum_{i=1}^N y_i = 0 \\ \hat{\alpha}^* &= \frac{\sum_{i=1}^N y_i}{N}\end{aligned}$$

Grading scheme: 50% of points for correct minimization problem. 75% of points if the final result is incorrect because of an error in calculations.

e) Replace $y_i = \alpha + u_i$ in the least squares estimator $\hat{\alpha}^*$ above:

$$\hat{\alpha}^* = \frac{\sum_{i=1}^N (\alpha + u_i)}{N} = \frac{N\alpha}{N} + \frac{\sum_{i=1}^N u_i}{N} = \alpha + \frac{\sum_{i=1}^N u_i}{N}$$

Applying the expectations operator to the $\hat{\alpha}^*$:

$E(\hat{\alpha}^*) = E\left(\alpha + \frac{\sum_{i=1}^N u_i}{N}\right) = \alpha$ since $E(u_i) = 0$. As $E(\hat{\alpha}^*) = \alpha$, $\hat{\alpha}^*$ is an unbiased estimator.

Given that $\hat{\alpha}^*$ is unbiased, it is necessary to show that its variance tends to zero as N tends to infinity, to prove that $\hat{\alpha}^*$ is also consistent:

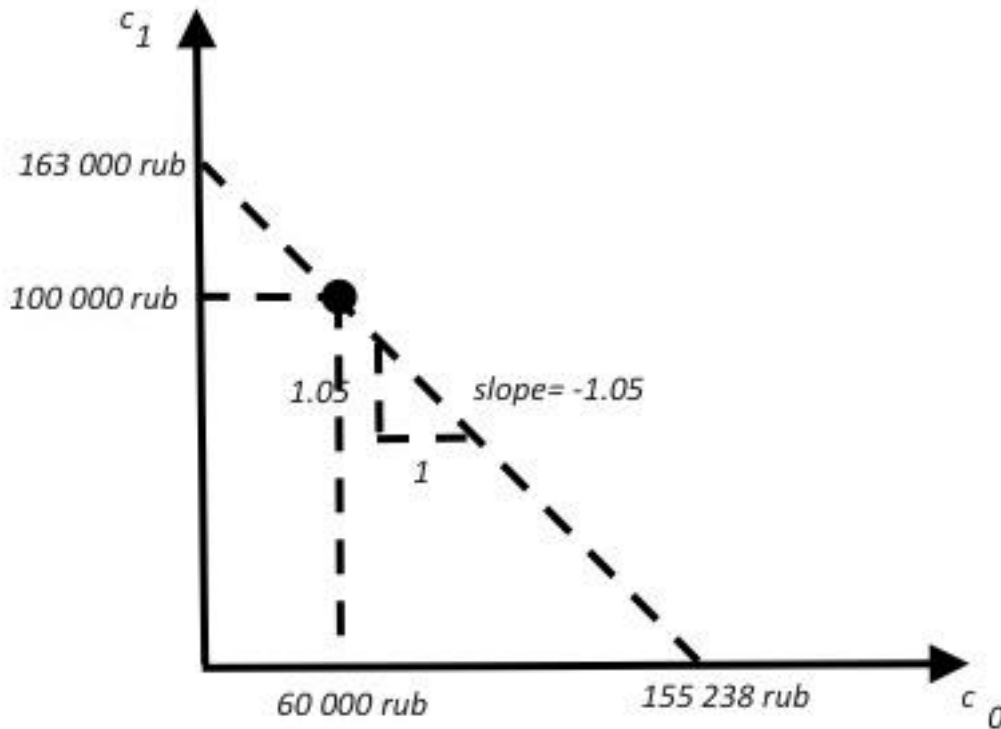
$$V(\hat{\alpha}^*) = E(\hat{\alpha}^* - \alpha)^2 = E\left(\frac{\sum_{i=1}^N u_i}{N}\right)^2 = \frac{1}{N^2} (u_1^2 + u_2^2 + \dots + u_N^2 + u_1 u_2 + u_1 u_3)$$

Since $E(u_i^2) = \sigma^2$ and $E(u_i u_j) = 0$ for all $i \neq j$, $V(\hat{\alpha}^*) = \frac{N\sigma^2}{N^2} = \frac{\sigma^2}{N}$, which obviously tends to zero as N tends to infinity. Therefore, $\hat{\alpha}^*$ is a consistent estimator.

Grading scheme: 50% of points for the full derivation of each property (unbiasedness and consistency). Probability limit can also be used to prove consistency of the estimator.

Answer for task 4

1.



The IBC is:

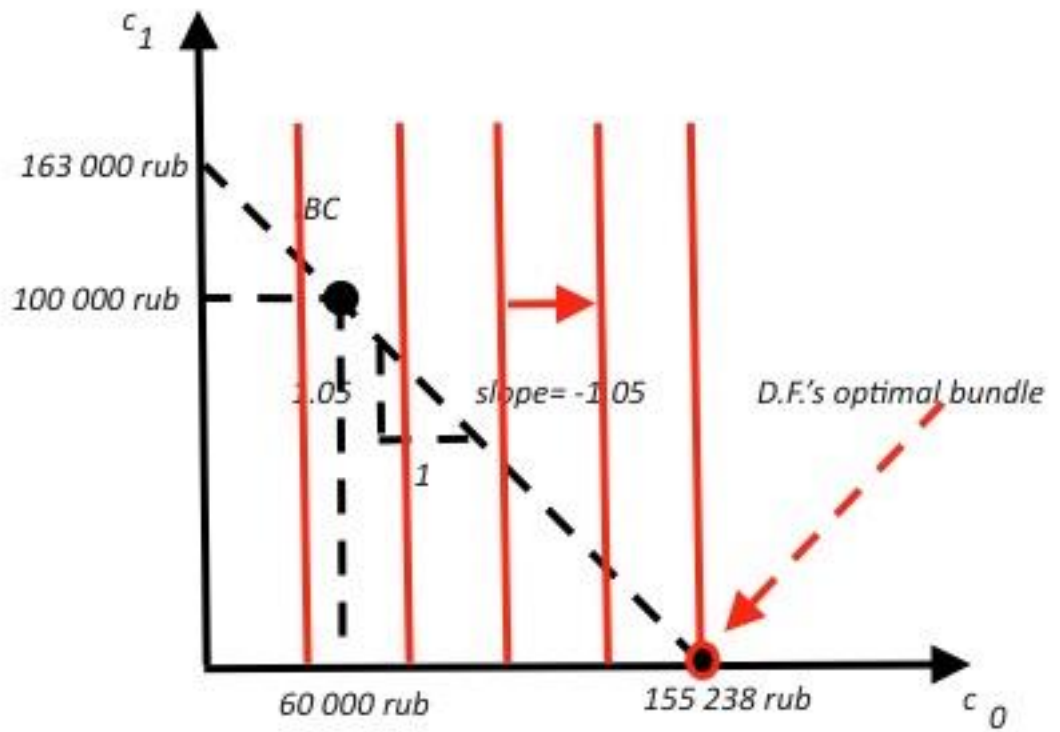
$$c_0 + \frac{c_1}{1+0.05} = 60\,000 + \frac{100\,000}{1+0.05}$$

1 point for the graph and 1 for the IBC.

2. The intertemporal budget constraint captures that the sons can either borrow against their future inheritance of 100 000 and get a maximum of $100\,000 / (1+0.05)$ rubles in the current period on top of the 60 000. Then they will have to use all of their future income to service the debt. On the other hand, they can put all of the 60 000 they get in the current period into the bank and get $60\,000(1+0.05)$ on top of the 100 000 they get in the second period. **(1.5 points)**

The slope of the budget constraint captures that to gain one more ruble in the current period, 1.05 rubles in terms of future consumption must be given up. Conversely, to gain 1.05 rubles of consumption in the future period, 1 ruble must be given up in the current period. **(1.5 points)**

3.



D.F. only cares about the current consumption. His optimal bundle is $c^ = (60\,000 + \frac{100\,000}{1+0.05}, 0)$. He is a borrower. In fact he borrows the maximum amount he can given his future income.*

1 point for the realization that the preference is not actually complicated. 1 point for the graph, 1 for the optimal bundle and 1 point for D.F. being a borrower.

4.

$$\alpha = 0.4$$

For this value, A.F. cares sufficiently about the Starez that he is willing to not borrow against his future income and provide his friend with some comfort. But he is not willing to forego current consumption and charity to facilitate this.

1 point for the graph, 1 point for realizing it is Leontief and $c_0 = (1 - \alpha)c_1$,

2 point for correct calculation and value of α . 1 point for the explanation of the value.

5. It is easy to see that there is a fundamental difference between the two brothers. While D.F. really only cares about the current period and thus transfers all of his future income into the current period via borrowing, A.F. has a more complex problem as he needs to weigh current versus future consumption (and charity). D.F. is always a borrower while A.F.'s status crucially depends on α . If it is above 0.4, he is a lender and transfers funds from the first to the second period). **(1 point)** If it is below, he prefers to have some more consumption (charity) in the current period than is possible with his current endowment. **(1 point)** For instance if $\alpha = 0$, he will surely borrow against future income. If it goes towards 1, he will save almost everything and spend it in the future on his friend. **(1 point)**
6. It depends if AF is a borrower or a lender. **(1point)** If he is a borrower, the income effect will be negative and he will consume less in both periods. If he is a lender (since he cares enough about Starez Sossima and α is above 0.4) his consumption goes up in both periods as the income effect is positive. **(1 point)** Since the proportions of consumption are fixed, there will be no substitution effect and he will proportionally increase or decrease his allocation. **(1 point)**

Answer for task 5

1. The discount rate is 0 and there are no taxes. The value of the firm V is given by

$$V = 0.5 \cdot (200 + 100) = 150$$

(1point)

The face value of debt is 150. If the high cash-flow is realized, the firm can repay its debt fully. In the other case, due to debt seniority, the debt holders get 100. Thus the value of debt today is:

$$0.5 * 100 + 0.5 * 150 = 125$$

(1 point for full explanation, in particular, students need to mention debt seniority, otherwise they receive 0.5)

The value of equity is $V - D = 25$.
(1 point)

2. The manager works in the interest of equity holders, thus he chooses the project that maximizes the cash-flows to equity holders. (1 point)

In project A, equity holders get $240 - 150 = 90$ in the high state, and 0 in the low state, thus an expected payoff of $0.5 * 90 = 45$. (1 point)

In project B, equity holders get $250 - 150 = 100$ in the high state, and 0 in the low state, i.e. on average 50. (1 point)

Thus equity holders prefer B, and the manager would choose B. (0.5 points)

The shareholders will finance the project since its NPV is positive: $0.5 * 100 - 30 > 0$ (1.5 points)

3. If only project A is available, and is financed through additional debt, the new debt holders will get 30 in each state since they have seniority over old debt holders. (1 point)

Given that the project cost 30 today, and the discount rate is 0, new debt holders break even and are willing to finance the project. (2 points, students can also say that new debt holders are indifferent, give half points if student correctly derive the state-dependent cash-flows but make a mistake elsewhere, or if the reasoning is correct but there is calculation mistake – the same applies to the next part of the question and the next question)

The old debt holders will get 150 in the high state, and 110 in the low state ($140 - 30$). This is an improvement relative to the baseline situation, where they get only 100 in the low state. Thus the old debt holders are willing to waive the seniority covenant. (3 points,)

4. First, note that if the manager has access to both projects (50% chance of this happening), he will invest in B, as shown in question (2). (1 point for this remark)

If the manager invests in project B, the cash-flows are large enough to repay new debt holders, so they are still willing to finance the project, even if the manager has access to both projects. (2 points)

Old debt holders, however, are reluctant to waiving the covenant because on average they get:

$$0.5*(150+110)/2+0.5*(150+20)/2=107.5 < 0.5*250=125$$

The left-hand side is the expected payment to debt holders of waiving the covenant when the manager has a 50% chance to access project B.

The second term of the left-hand side, in particular, is the expected payment to debt holders when project B is available.

The right-hand side is the expected payment of not waiving the seniority covenant (i.e. as in question (1), there is no investment in a new project).

Old debt holders thus refuse to waive the covenant.

(3 points in total for this, 1 point for each element: the payoff if covenant is waived and one project is available, the payoff if covenant is waived and both projects are available, 1 point for payoff if covenant is not waived)

Good student premium: +1 point if they mention the following:

This exercise illustrated the risk-shifting problem. In firms with both debt and equity, managers working on behalf of equity holders have an incentive to invest in lower NPV and riskier projects, since they are protected by limited liability in the downside. This is at the expense of current debtholders, who can limit this behaviour by imposing seniority covenants.

THE TASK'S TOTAL SCORE MUST BE ROUNDED UP TO THE NEXT INTEGER
(NO DECIMAL SCORE FOR THE TASK)

End of file