ICEF 2019 Olympiad – Solutions

General grading guidelines

These are 5 original tasks created by aninternational academic committee especially for the purpose of this Olympiad. Each task has been created by a specialist in the area, edited by the academic director of the Olympiad and then blindly reviewed by another specialistindependently. These are open ended tasks. The answers will be marked blindly at the discretion of the grading team in a manner uniform for all candidates.

Guidelines

1. Full credit should be reserved for answers that demonstrate mastering the material and the explanation is sufficient and flawless at the discretion of the grader.
2. Partial credit may be given to insufficient answers towards the correct direction at the discretion of the grader.
3. Minimal credit may be given to answers not towards the correct direction, which however indicate that the candidate has some limited knowledge on the subject at the discretion of the grader.
4. Zero credit should be given to answers that are wrong, irrelevant, make no sense, or have random correct elements, at the discretion of the grader.

Any indication of failure to master the material should result to a reduction of credit. For example, if a candidate answers the question perfectly and then attaches an additional part which is irrelevant or wrong, the grader mayremove points because the redundancy indicates failure in understanding.

Appeals

Candidates have the rightto appeal to the decisions made by the graders according to the HSE Olympiad regulations.

The academic director of the Olympiad will decide if an appeal will be forwarded to the members of the committee for re-grading or it will be terminated as lacking merit. Appeals will be forwarded for re-grading only when one of the following occurs:

1. <u>The candidate sufficiently justifies</u> that the grader has made a specific mistake grading a specific part of the answer.
2. <u>The candidate sufficiently proves</u> that there is a misspecification in some part of the question that allows the candidate's answer to be interpreted as a partially or fully correct response.
3. <u>The candidate sufficiently proves</u> that there is a mistake in a specific part of the question such that there is no correct response.
4. <u>The candidate sufficiently demonstrates</u> that the given answer is also (partially) correct, even though it is not included in this answer key.

Appeals that are insufficiently justified; or do not fall under one of the above 4 cases; <u>or are not in English</u>; or are unclear; or are just asking for higher grades, will be terminated by the academic director of the Olympiad with the indication: "The appeal has no merit (# reason #)".

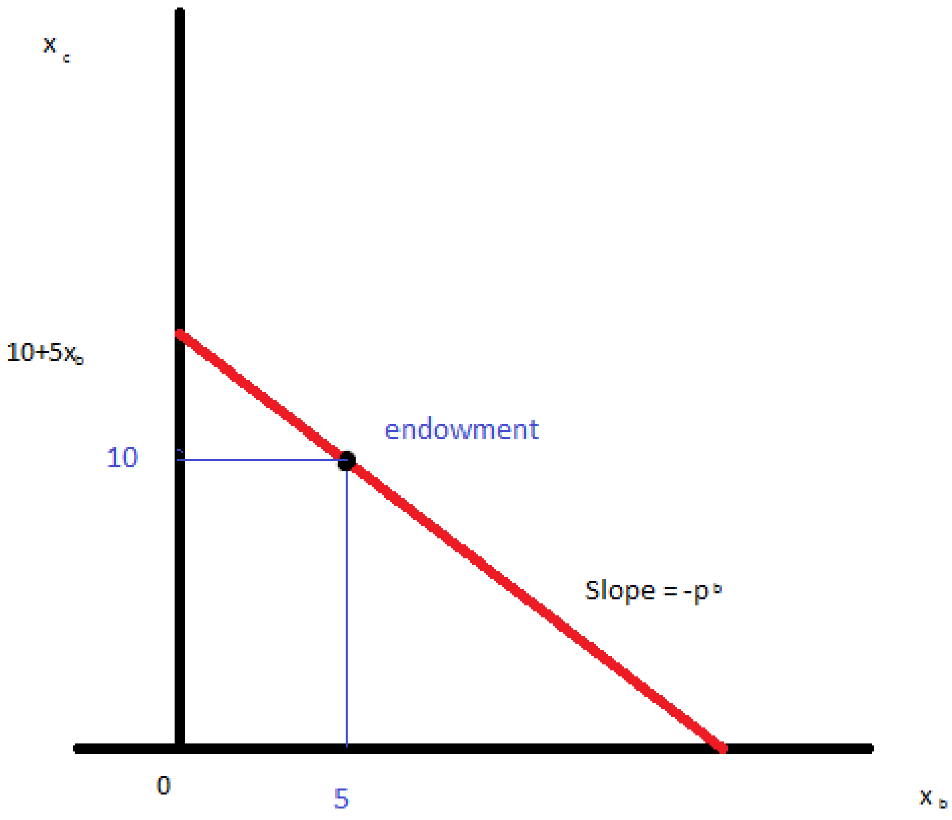
If the appeal is judged as reasonable by the academic director, a committee member will be asked to re-grade the question. In this case the candidate is possible to receive a higher or a lower or the same gradein comparison to that received in the first degree.

Question 1

a.

The budget constraint is $p_b x_b + x_c = p_b 5 + 10$ (1 point).

The budget constraint as a curve is $x_c = 10 + 5p_b - p_b x_b$:



(2 pt for graph) (do not deduct points if instead of slope they labelled horizontal intercept

correctly)

b.

Daniil chooses his bundle to maximize $5 + 3\ln(x_b) + 3\ln(x_c)$ subject to $p_b x_b + x_c = 10 + 5p_b$.

(1 point)

By substitution (or similar answer using Lagrange), Daniil maximizes $F(x_b) = 5 + 3\ln(x_b) + 3\ln(5p_b + 10 - p_b x_b)$ with first order condition:

$F'(x_b) = 3/x_b - 3p_b/(5p_b + 10 - p_b x_b) = 0$

(0.5 point each for problem and FOC)

Solving we get $x_b = 2.5 + 5/p_b$ and $x_c = 2.5p_b + 5$

(1 point)

The second order condition verifies that it is a maximum:

$F''(x_b) = -3/x_b^2 - 3p_b^2/(5p_b + 10 - p_b x_b)^2 < 0$

(1 point)

c.

Objective. Alexei chooses his bundle to maximize $4x_b + x_c$ subject to $p_b x_b + x_c = 10 + 15p_b$.

(1 point)

Demands. By substitution (or similar answer using Lagrange), Alexei maximizes $F(x_b) = 4x_b + 15p_b + 10 - p_b x_b$ or $F(x_b) = 4x_b + 15p_b + 10 + (4 - p_b)x_b$.

(0.5 point)

If $p_b < 4$ then $x_b = 15 + 10/p_b$ and $x_c = 0$. If $p_b > 4$ then $x_c = 15p_b + 10$ and $x_b = 0$. If $p_b = 4$ then Alexei is indifferent to any bundle that satisfies his budget constraint.

(1.5 point for this discussion)

Price. Since by assumption each agent is consuming a positive amount of each good, the Walrasian equilibrium price must be $p_b^* = 4$.

(1 point)

Allocations. By substitution, Daniil's allocation is $(x_b^*, x_c^*) = (2.5 + 5/(4), 2.5(4) + 5) = (3.75, 15)$ and Alexei's allocation is given by the leftovers $(x_b', x_c') = (20 - (3.75), 20 - (15)) = (16.25, 5)$.

(2 points: 0.5 for each)

d.

No. (0.5 pt)

Daniil's new demand is chosen by maximizing $x_b^2 + x_c^2$ subject to $p_b x_b + x_c = 10 + 5p_b$. (0.5pt)

Finding demands. By substitution, Daniil maximizes $F(x_b) = x_b^2 + (10+5p_b - p_b x_b)^2$ which gives FOC$F'(x_b) = 2x_b - 2(10+5p_b - p_b x_b) = 0$. (0. 5 point)

However, the second-order condition $F'(x_b) = 2 + 2p_b > 0$ indicates that the solution is a corner solution. (0.5 point)

Daniil's new demand is described as follows. If $p_b<1$ then $x_b=15p_b+10$ and $x_c=0$. If $p_b>1$ then $x_c=15p_b+10$ and $x_b=0$. If $p_b=1$ then Daniil is indifferent between these two bundles.

(1 point)

If $p_b=4$ then Daniil demands $x_c=5(4)+10= 30$ chocolate. However, the supply of chocolate is 20, so the price does not clear the market for chocolate.

(1 point)

e.

Answer about price and uniqueness. The price for beer that clears the chocolate market is $p_b=2$ and this is the only such price. (0.25 point) If $p_b>4$ then Daniil demands more than 20 kg of chocolate while Alexei demands a positive number of (kg of) chocolate. (0.25 point) If $2<p_b<4$ thenDaniil demands more than 20kg of chocolate while Alexei demands 0kg of chocolate. (0.25 point) If $p_b<2$ then Daniil demands less than 20kg of chocolate and Alexei demands 0 chocolate. Therefore, the only price that clears the chocolate market is $p_b=2$ and by Walras's Law it is also the only price that clears the beer market. (0.25 point)

Daniil demands $(x_b^*,x_c^*)=(0,5(2)+10)=(0,20)$ and Alexei demands $(10/(2)+15,0)=(20,0)$. (2 pts for allocations: 0.5 pt per good per agent)

Question 2

a.
The maximization problem is

$$\max_{K,N} f(K,L) - cK - wN$$

The first-order conditions give

$$f_K = c, \text{and } f_L = w$$

1 point for writing the maximization problem, 1 point for FOC.

b.
Starting from the function f and differentiating, we get

$$dY = f_K dK + f_L dL = f_K dK + f_L(Ndh + hdN)$$

(1 point)

Dividing by Yand rearranging terms, we get

$$\frac{dY}{Y} = f_K \frac{dK}{K}\frac{K}{Y} + f_L \frac{hN}{Y}\left(\frac{dh}{h} + \frac{dN}{N}\right)$$

(1 point)

Then using the FOC of the previous question, writing $\alpha = cK/Y$ and $1\text{-}\alpha = wL/Y$, and using notations g_Y, $g_K$and g_N, we obtain $g_Y = \alpha g_K + (1\text{-}\alpha)(g_N + g_h)$.
(1 point)

This gives:

$$g_h = \frac{1}{1-\alpha}(g_Y - \alpha g_K - (1-\alpha) g_N)$$

So the growth rate of the index quality is proportional to Solow residual.
(1 point)

Students do not have to use the same notation.

c.
The program of the firm is

$$\max_{K,N_u,N_s} f(K,L) - cK - w_u N_u - w_s N_s$$

(1 point)
First-order conditions

$$f_L e_u G_u = w_u$$
$$f_L e_s G_s = w_s$$

(1 point)

These conditions imply that

$$f_L(e_u N_u G_u + e_s N_s G_s) = w_u N_u + w_s N_s$$

(0.5 point)

Given that G is homogeneous of degree one, $e_u N_u G_u + e_s N_s G_s = G = L$.

(1 point – derivation of this property not needed)

The weighted average salary is

$$\overline{w} = \frac{w_u N_u + w_s N_s}{N}$$

(0.5 point)

So replacing the numerator, we get

$$\overline{w} = \frac{L f_L}{N} = h f_L$$

(1 point)

i) Starting from $h = \frac{L}{N} = \frac{G}{N} = G\left(\frac{e_u N_u}{N}, \frac{e_s N_s}{N}\right)$, where we used the homogeneity of G, (1 point) we differentiate

$$dh = \frac{N_u}{N} G_u de_u + \frac{N_s}{N} G_s de_s + e_u G_u d\left(\frac{N_u}{N}\right) + e_s G_s d\left(\frac{N_s}{N}\right)$$

(1 point)

From the FOCs, we have $e_u G_u = w_u {f_L}^{-1}$ and $e_s G_s = w_s {f_L}^{-1}$
Substituting, we get:

$$dh = {f_L}^{-1}\left[\frac{N_u w_u}{N}\frac{de_u}{e_u} + \frac{N_s w_s}{N}\frac{de_s}{e_s} + w_u d\left(\frac{N_u}{N}\right) + w_s d\left(\frac{N_s}{N}\right)\right]$$

(1 point)

Since from the previous question,

$$h = \frac{\overline{w}}{f_L}$$

$$\frac{dh}{h} = \frac{N_u w_u}{N\overline{w}}\frac{de_u}{e_u} + \frac{N_s w_s}{N\overline{w}}\frac{de_s}{e_s} + \frac{w_u}{\overline{w}} d\left(\frac{N_u}{N}\right) + \frac{w_s}{\overline{w}} d\left(\frac{N_s}{N}\right)$$

(1 point)

ii) Through this relationship we have decomposed the change in quality into 2 components. The first one (first two terms) is related to the change in the technical coefficients affecting the two types of labor. That remains unexplained. The second one is related to the change in the composition of the workforce. This component can help partly explain Solow residual. The improvement in the quality of the workforce accounts for part of the residual.
(2 points)

iii) This approach relies on the premise that the factors are remunerated at the competitive level. In reality, the labor market is far from being perfectly competitive, which will distort the calculation of Solow residual. Other frictions such as payroll taxes would create a wedge between salary and marginal product. More generally, firm/ market frictions can create such wedges.
(2 points. Various answers are possible)

Question 3

a.
Approximate interpretation: an extra year of education is associated on average with 8% increase in wages; the average salary for men is 21.5% higher than for women keeping education constant (this is bad approximation); an extra year of education is associated with 5.2% higher increase in wages for men than for women. Precise interpretation: an extra year of education is associated on average with 100%*$e^{0.08} \approx$ 8.3% increase in wages; the average salary for men is 100%*$e^{0.215} \approx$ 24% higher than for women keeping education constant; an extra year of education is associated with 100%*$(e^{0.097} - e^{0.045}) \approx$ 5.6% higher increase in wages for men than for women.

1 point for interpretation of the first coefficient, 1 point for the second and 2 points for the third. Both the approximate and the precise interpretations are acceptable (it may be that students do not have appropriate calculators to calculate exponentials and that they mention it in their answer sheet. For the second coefficient the approximation is very inaccurate but grader should not take points off). Give at most 2 points if the interpretation is causal (e.g., "an extra year of education increases the wages by 8%"). Give at most 1 point if the answer is about unit changes in wages and not the percentage changes. Give 0 points out of 1 for the interpretation of the second coefficient if they don't write "keeping education constant".

b.
From Model A, since $R_A^2 = 1 - \frac{RSS_A}{TSS}$, $TSS = \frac{RSS_A}{1-R_A^2} = \frac{66.12}{1-0.14} \approx 76.88$. Then $R_B^2 = 1 - \frac{RSS_B}{TSS} \approx 1 - \frac{57.65}{76.88} \approx 0.25$.

1 point for the formula for R^2, 1 for correctly computing TSS, and one for correctly computing R^2 for Model B.

c.
H_0: $\beta_3 = 0$. The test statistic is equal to $t = \frac{\hat{\beta}_3}{s.e.(\hat{\beta}_3)} \approx \frac{0.052}{0.022} = 2.36$. From the table we can tell that $p_{value} = 2(1 - 0.9909) = 0.0182$. Since $0.0182 > 0.01$, we fail to reject the null at 1% significance level.

1 point for correctly stating the hypothesis.1 point for the formula for t-stat, and 1 for correctly computing it.1 point for correct p-value (0 if done for one-sided hypothesis), and 1 point for the correct conclusion. 0 points if they test the hypothesis by comparing the t-stat to the critical values from the table. There might be some variation in the p-value (depending on how they round the t-stat), but it should be between 0.0178 and 0.0182. Any answer within this range should be considered correct.

d.
While it is true that R^2 is larger for Model B, it does not mean that Model B is better, because R^2 never decreases when we add more variables to the model. What a researcher could have done is looked at adjusted R^2, which controls for this property of R^2 . The adjusted $R^2$for Model A is $adjR_A^2 = 1 - (1 - R_A^2)\frac{N-1}{N-2} \approx 0.136$, and for Model B it is equal to $adjR_B^2 = 1 - (1 - R_B^2)\frac{N-1}{N-4} \approx 0.24$. So, according to the adjusted R^2, Model B is better.

2 points for stating that R^2 increases if we add more variables to the regression and so it is wrong to compare models with different number of regressors. 1 point for explaining how to compare the models (based on adjusted R^2, the F-test, or some information criteria: AIC or BIC).

e.

This is the test for $H_0: \beta_2 = \beta_3 = 0$. It can be performed with the help of the F-test. For this hypothesis, Model A is the restricted model, and Model B is the unrestricted one. Hence,

F=$\frac{\frac{R_{UR}^2 - R_R^2}{q}}{\frac{1-R_{UR}^2}{N-k}} = \left(\frac{\frac{RSS_R - RSS_{UR}}{q}}{\frac{RSS_{UR}}{N-k}}\right) = \frac{\frac{0.25-0.14}{2}}{\frac{1-0.25}{236}} \approx 17.31$. The F-statistic has the F-distribution with (2,236) degrees of freedom. Since this value is larger than 1% critical value for F(2,120) provided in the table, it is also larger than the 1% critical value for F(2,236), and so we can reject the null at any reasonable significance level.

2 points for correct expression of F-statistic and correct values of q, k, and R^2. (1 point only, if k is wrong, 1 point only, if q is wrong, 1 point only, if wrong R^2 is used in the denominator; 0 points if two or more of these mistakes are made.) 1 point for correct calculation, 1 point for checking the table correctly and 1 for correct conclusion (0 points if the conclusion is made only at 5% level).

Question 4

a.
Consider first how to induce the risk-neutral agent to participate in the contract when effort is observable.

In case of high effort, the producer must pay w such that $w - 6 \geq 1$, i.e. $w \geq 7$.
Since the effort is observable, the producer can simply pay the agent 7.
(1.5 points)

Under low effort, the producer must pay w such that $w - 2 \geq 1$, i.e. $w \geq 3$.
Since the effort is observable, the producer can simply pay the agent 3.
(1.5 points)

b. Second, let's determine when it is optimal for the producer to induce high effort. The expected gain of the producer is

- $\frac{2}{3}x - 7$ under high effort (1 point)
- $\frac{1}{3}x - 3$ under low effort (1 point)

So inducing high effort is optimal if $x \geq 12$. For $x \geq 12$, the producer offers the first contract, otherwise he offers the second one.
(1 point)

c.
Under high effort, the risk-neutral agent participates if

$$\frac{2}{3}w_s + \frac{1}{3}w_n - 6 \geq 1$$

(1 point)

The agent prefers high to low effort if

$$\frac{2}{3}(w_s - 6) + \frac{1}{3}(w_n - 6) \geq \frac{1}{3}(w_s - 2) + \frac{2}{3}(w_n - 2)$$
$$i.e., \qquad w_s - w_n \geq 12$$

(1 point)

Given risk-neutrality and limited liability, the producer can set $w_n = 0$.
(1 point)

Under low effort, the agent participates if

$$\frac{1}{3}w_s + \frac{2}{3}w_n - 2 \geq 1$$

As in the previous case, the producer can set $w_n = 0$, so $w_s \geq 9$. (the incentive constraint to exert low effort places an upper bound of 12 on w_s, but it is not needed to recall it)
(1 point)

d.
Under high effort, the two constraints imply $w_s \geq 3/2$and $w_s \geq 12$. So the producer will optimally set $w_s = 12$ (0.5 point).

The expected profit for the producer under high effort is thus $2/3x - 8$. (1 point)

Under low effort, $w_s \geq 3$, thus the producer will optimally set $w_s = 3$. (0.5 point)

The expected profit for the principal under low effort is $1/3x - 3$. (1 point)

Thus, the optimal contract is the first for $x \geq 15$ and the second for $x < 15$.
(1 point)

e.
Suppose that agent 2 exerts high effort. Take the point of view of agent 1. Her probabilities of success under high effort and low effort are the same as before:

If agent 1 exerts high effort, he sells with probability 1/3+1/3=2/3. (1 point)

If he exerts low effort, he sells with probability 2/9+1/9=1/3. (1 point)

Agent 2 is symmetric. (2 points)

Thus, the optimal incentivizing contract that rewards the agents independently and induces high effort of each agent is the one derived at point 2. (2 points)

Question 5

a.
From the perpetuity formula, the value of the unlevered firm (equal to unlevered equity value) is

$$V_u = E_u = \frac{1}{0.12} = 8.33m$$

(1 point for perpetuity calculation, 0.5 point for explaining that $V_u = E_u$)

There are 0.5m shares outstanding, thus the share price is

$$\frac{8.33}{0.5} = \$16.66$$

(0.5 point for the share price)

b.
By Modigliani-Miller Proposition 1, the firm value does not change, $V_u = V_l = 8.33m$. Here, there are no taxes and no frictions, so MM propositions hold.
(0.75 point for answer, 0.25 for mentioning the absence of taxes and frictions)

This implies that the value of levered equity, denoted E_l, is

$$E_l = V_l - D = 8.33 - 0.8 = 7.53m$$

(1 point)

The share price after the dividend payment is \$15.06. First, the firm raises cash through the debt issuance. At this point, there is no change in equity or share price. Then the firm uses the cash to pay dividend, and the price falls by exactly this amount (0.8/0.5=1.6).
(0.5 point for calculation, 0.5 point for explanation)

Other valid interpretation: since assets have not changed, but there is debt in the capital structure, it must be that the value of equity is small by exactly this amount. Similarly, the stock price is smaller by this amount divided by the number of shares.

The rate of return on levered equity r_e is given by Modigliani-Miller Proposition 2:

$$r_e = r_u + \frac{D}{E_l}(r_u - r_d) = 12\% + \frac{0.8}{7.53} * (12\% - 5\%) = 12.74\%$$

(1 point, only 0.5 point if student uses $E_u$instead of E_l in the calculation)

The return on equity rises compared to the unlevered case. The risk of the assets is unchanged. But the firm is now portfolio of debt and equity. Since debt is less risky than overall portfolio, equity must be riskier than the whole.
(1 point)

Note: as an alternative method, one can find the discount rate that is consistent with a cash flow to shareholders of 0.96 (operating cash-flow minus interest expense, i.e. 1-0.05*0.8, 0.5 point for this calculation) and a value of levered equity E_l =7.53, since $E_l = \frac{\text{CF to levered equity}}{r_e,}$.
This gives

$$r_e = \frac{0.96}{7.53} = 12.74\%$$

(0.5 point for this calculation, rest of the points as above)

c.
Due to Modigliani-Miller 1, the firm value is again unchanged (1 point).

To calculate the new share price, the additional complication is that the number of shares changes due to share repurchase.

Denote P the new share price and x the new number of shares, after the repurchase. Then x and P are given by:

$$0.5 - \frac{0.8}{P} = x$$
$$P = \frac{7.53}{x}$$

The first equation states that the new number of shares is the initial number (0.5m) minus the number of shares bought thanks to the money raised in the debt market, 0.8m at a price P, i.e. 0.8/P.

The second equation states that the new price is equal to the new equity value divided by the new number of shares. Solving, wet get P=16.66 and x=0.452.

(2 points for exact system, half point if student writes something close but not exactly right)

The price is unchanged, since otherwise there would be an arbitrage.
The wealth of equity holders has not changed. Those who kept their shares have the same wealth since the price is unchanged, and those who sold their shares received $16.66 per share, which was their initial price. Thus, as stated by MM, the firm's financial policy is irrelevant.
(2 points for explanation, adjust depending on clarity and quality)

d.
First, let's recalculate the value of unlevered equity and the price of unlevered stock.

$$V_u = E_u = \frac{1 * (1 - 0.34)}{0.12} = 5.5m$$

(0.5 point for calculation)

So the share price of the unlevered firm is

$$P_u = \frac{5.5}{0.5} = \$11$$

(0.5 point for stock price)

The present value of the debt tax shield is

$$PV(DTS) = \sum_{t=1}^{\infty} \frac{Interestexpense * t_c}{(1 + r_{dts})^t}$$

(0.5 point)

Since debt is a perpetuity, $Interestexpense = r_d D = 0.05 * 0.8$
(0.5 point for this argument).

Since $r_d = r_{dts}$,
(0.5 point for this argument)

we can simplify PV(DTS) to

$$PV(DTS) = \sum_{t=1}^{\infty} \frac{r_d D * t_c}{(1 + r_d)^t} = D * t_c$$

(0.5 point, students may write directly the perpetuity formula)

Thus the value of the levered firm is

$$V_l = V_u + PV(DTS) = 5.5 + 0.34 * 0.08 = 5.772$$

The value increases due to the tax shield.
(1 point for calculation)

The value of levered equity is

$$E_l = V_l - D = 4.972$$

(0.5 point)

The stock price after the debt issuance is 4.972/0.5=9.944. The stock price goes down due to the dividend payment.
(0.5 point)

(However, unlike in b., the wealth per share of a shareholder increases:
Before it was $11 per share
After it is 9.944+0.8/0.5=11.544

Thus shareholders capture the full benefit of the tax shield: 0.8*0.34/0.5=0.272/0.5=0.544. Explanation not required, but students explaining this should get higher marks)

e.
The fixed amount of taxes of 0.04M every year is similar to paying interest in that amount every year. Hence taxes are the same as taking on perpetual debt of 0.8M at 5% interest rate. Hence the cost of equity increases to 12.74%, for the same reason as in b.

(3 points for explanation + result. If students fail to realize that the numbers are the same as in b, but explain that a fixed amount of tax every year is similar to taking on debt, they should get up to 2.5 points, depending on the quality of the answer)