Олимпиада студентов и выпускников «Высшая лига» – 2020 г.

Направление «Финансовая экономика»

Профиль:
«Финансовая экономика / Financial Economics» **КОД - 130**

Время выполнения задания - 180 мин, язык – английский.
Максимальное количество баллов- 100.

Examination Guidelines

- The exam consists of 5 tasks. Solve all tasks. The examination time is 180 minutes.
- The exam is graded on a 100-point scale. Numbers in brackets indicate the points awarded for each problem.
- Write your answers in the booklet provided to you by the examiners.
- You can solve the tasks in any order but you must label each task and sub-task clearly and sufficiently. Use a separate page for each task. You are not allowed to detach sheets from the booklet.
- Answer all tasks in English. Graders will ignore any Russian text.
- You may use the last page of your booklet as scrap paper.
- Crossed out writing will not be considered by the grader.
- Use legible hand writing. The grader will ignore any illegible parts of your paper.

Examination Rules

- You are required to follow all instructions given by the examiners.
- Talking is not allowed under any circumstances.
- During the exam you are allowed to have on your desk two pens (black or blue), a drink and a non-programmable scientific calculator. You are not allowed to bring any written or printed materials, mobile phones or other electronic devices into the examination room.
- Detection of any electronic communication device on you will constitute cheating even if the device is off.
- The proctors of the exam are not authorized to answer any questions.
- Exam participants are not allowed to leave the examination room until ready to turn in their work.

I have read and understood the examination rules. I will not cheat, copy or use unauthorized materials or devices.

Signed: ________________________

Национальный исследовательский университет «Высшая школа экономики»

Problem 1: asymmetric information [20 points]

An entrepreneur has an investment opportunity that requires investment I = 100. He has own funds of size $A < I$, meaning that he needs to raise the remaining funds in the market. After the funds have been invested, in one period, the entrepreneur generates a verifiable cash flow, available for distribution to investors, and derives a non-verifiable *private* benefit. The life of the project then ends, there are no further cash flows or private benefits.

The entrepreneur can be one of the two types: "good" and "bad", with *equal* probabilities. These probabilities are common knowledge, but the entrepreneur's type is his private information. A good type generates the expected cash flow of 120 and derives zero private benefit. A bad type generates the expected cash flow of 60 and derives the private benefit of 60.

Everybody is risk-neutral and fully rational. There is no discounting. The financial market is perfectly competitive. Formally, the fund-raising process is as follows. The entrepreneur offers a financial security: an equity share in case of equity financing or a face value of (zero-coupon) debt in case of debt financing. Observing this, the market forms beliefs about the entrepreneur's type and values the security accordingly. If the value is at least I-A, the funds are provided (i.e., the offered security is bought). Otherwise, no financing occurs. Any funds raised in excess of I-A are pocketed by the entrepreneur.

Definitions:

- An equity share α entitles its holder(s) to the share α of the cash flow. If the entrepreneur sells share α, he is left with share (1-α) of equity.
- A zero coupon debt with face value F entitles the debtholder to receive the fixed amount F out of the realized cash flow. If the realized cash flow is less than F, the debtholder receives the whole cash flow (due to the limited liability of the entrepreneur).

In parts (a) and (b) assume that the entrepreneur *is not allowed* to issue debt.

Throughout the problem, assume that upon observing an offer which is not expected in equilibrium the market beliefs that the type is bad.

a) [6 pts] Suppose A=0. Is there an equilibrium in which both types of the entrepreneur raise the necessary funds by issuing:
   i. Different equity shares?
   ii. The same equity share?

b) [8 pts] Now assume A=50. Answer questions i. and ii. of the previous part.

c) [3.5 pts] Keep assuming A=50. Suppose now that the entrepreneur is allowed to issue debt. Suppose the debt with face value (promised repayment) of 50 is risk-free regardless of the entrepreneur's type. What will each type of the entrepreneur issue in equilibrium?

d) [2.5 pts] Return to the assumption of A=0. Suppose now the bad type is characterized not by private benefit extraction but by higher riskiness of the cash flow. Specifically, assume that the good type generates the cash flow of 120 with certainty, while the realization of the bad type's cash flow can be 70 or 170 with equal probabilities. As before, the entrepreneur's type is his private information. At the time of financing, the bad type does not know whether the cash flow will be high or low. The probabilities of each type as well as the probabilities of the low and high cash realizations in the bad entrepreneur's project are common knowledge.

Both equity and debt are allowed to be issued. The entrepreneur is protected by limited liability, and there are no bankruptcy costs.

Show that there exists an equilibrium in which the good type issues only equity, and the bad type issues only debt.

Problem 2: econometrics [20 points]

A lecturer teaches a course on mathematical methods in economics. This course consists of 30 hours of lectures and 48 hours of class sections. The lecturer is interested in factors that determine academic performance of students in his course. He collected data for two student groups, G1 and G2, and constructed the following three models:

Model A: $TS_i = a_0 + a_1\, Lectures_i + a_2\, CS_i + \varepsilon_i$

Model B: $TS_i = b_0 + b_1\, Lectures_i + b_2\, CS_i + b_3 G_i + u_i$

Model C: $TS_i = c_0 + c_1\, Lectures_i + c_2\, CS_i + c_3 G_i + c_4\, G_i \cdot CS_i + \epsilon_i$

where TS_i is student i's exam grade on scale from 0 – 100 (not necessarily an integer), $Lectures_i$ is the percentage of lectures that student i attended during the semester, CS_i is the percentage of class sections attended by student i, and G_i is a dummy variable equal to 1 if student i is from group G1 and 0 otherwise. $\varepsilon_i, u_i, \epsilon_i$ are the error terms, each of them is independent across students and normally distributed.

The estimation results are reported below (in parenthesis are the standard errors, and RSS stands for the residual sum of squares):

	Model A	Model B	Model C
intercept	25.974 (5.388)	19.645 (2.703)	22.766 (2.915)
Lectures	0.048 (0.098)	0.151 (0.049)	0.184 (0.049)
CS	0.517 (0.098)	0.319 (0.051)	0.218 (0.066)
G		26.103 (2.191)	15.727 (4.970)
G*CS			0.177 (0.077)
R^2	0.55	?	?
RSS	9394	2223	1979
N	48	48	48

a) [2 pts] Interpret the coefficients on Lectures, CS and G in Model B.

b) [2 pts] Test the overall significance of Model A at 99% confidence level: write the null and the alternative hypotheses, compute the test statistic, perform the test and write the conclusion. Refer to tables in appendix for distribution functions for t-distribution and F-distribution.

c) [4 pts] Compute the R2 for Model B and Model C. Which model (B or C) explains the data better? Support your answer by a formal argument.

d) [3 pts] Suppose that a student from group G2 (who is involved in a lot of social activity and does not attend all the lectures and class sections) wants to spend additional 1.5 hours to study for this course to improve the expectation of his grade. He needs to choose whether to attend an additional lecture or an additional class section. Using the estimation results for Model C, help the student to decide what to do. Also, what would be your suggestion if the student were from Group G1?

e) [4 pts] Test the hypothesis that the effect of class sections attendance is the same for both groups. State the null, the alternative, calculate the approximate p-value of the test and make the conclusion.

f) [5 pts] The lecturer knows that many students taking his course have graduated from mathematical classes, but he does not have the data on each particular student. Does he have an omitted variable bias in Model A? If yes, explain why and state the estimator(s) of which coefficient(s) are biased and what the sign of the bias is; if no, explain why.

Problem 3: finance [20 points]

Consider an economy with three dates, $t = 0,1,2$ and a continuum of agents of measure 1. There is a single divisible, non-storable good. Each agent starts with a unit of the good as initial endowment.

Because the good cannot be stored, agents can only transfer wealth across periods by investing their good (or part of it). The investment technology in the economy yields a sure net return r_1 from time 0 to time 1 and a stochastic net return $r_2 \geq -1$ from time 1 to time 2. Agents cannot observe directly r_2 but they observe a public signal $s \in [0,1]$ at time 1, with $f: s \mapsto E[r_2|s]$ increasing and $f(1)$ very large.

Agents choose non-negative consumptions C_t to maximize expected utility. They have preferences represented by $U(C_1, C_2) = C_1 + \frac{1}{1+\rho} C_2$, where $\rho > 0$. At time 1, the public signal s is observed before the consumption decision C_1 is made.

a) [4 pts] Write the maximization problem of an agent and prove that the optimal (symmetric) allocation is given by

$$C_1 = 1 + r_1 \text{ and } C_2 = 0 \text{ if } 1 + \rho > E[1 + r_2|s],$$
$$C_1 = 0 \text{ and } C_2 = (1 + r_1)(1 + r_2) \text{ if } 1 + \rho < E[1 + r_2|s].$$

Suppose now that the agents cannot operate the technology themselves. Instead, they all invest their endowment in a bank which has access to the technology. In exchange of their investment, the bank offers them a demand deposit contract that promises a net return $r_D > \rho$ from time 0 to time 1, and also r_D from time 1 to time 2.

When the bank fails to pay the promised amounts in full, its assets are distributed in proportion to the depositors' claims. At time 1, after observation of s, agents simultaneously decide whether to withdraw their funds or stay invested in the bank. We do not consider partial withdrawals: each individual agent either withdraws all her deposits or none at date 1.

Let $\widehat{d_1}$ denote the total amount effectively withdrawn at time 1. Let $\delta(\widehat{d_1}, s)$ denote the expected gross return on deposits from time 1 to time 2, given the public signal s.

b) [6 pts]
i) Explain why the maximal possible value of $\widehat{d_1}$ is $\min\{1+r_1, 1+r_D\}$.
ii) Explain why the fraction of creditors who remain invested at time 1 is given by $1-\frac{\widehat{d_1}}{1+r_D}$, in case the bank does not default at time 1.
iii) In case of no default at time 1 but default at time 2, how much does a creditor who remained invested get?
iv) Conclude that, for any $\widehat{d_1} < \min\{1+r_1, 1+r_D\}$, we have
$\delta(\widehat{d_1}, s) = E\left[\min\left\{\frac{(1+r_1-\widehat{d_1})(1+r_2)}{1+r_D-\widehat{d_1}}, 1+r_D\right\} | s\right]$.

c) [4 pts]
i) Prove that $\delta(\widehat{d_1}, s)$ is (weakly) increasing in $\widehat{d_1}$ if $r_1 > r_D$ and (weakly) decreasing in $\widehat{d_1}$ if $r_1 < r_D$.
ii) What does it imply in terms of strategic complementarity (resp. substitutability) of the withdrawing decisions, i.e.: when does more other agents withdrawing give incentives to withdraw as well (resp. to not withdraw)?

d) [6 pts] Assume that $r_1 < r_D$. The decision to withdraw or stay invested at time 1 constitutes a static game between the depositors.
i) Show that there always exists a pure strategy equilibrium in which all agents withdraw.
ii) Prove that there exists a threshold s^* such that for $s > s^*$, the game has a second symmetric equilibrium in pure strategies, in which all agents remain invested.
iii) When do the equilibria of questions i) and ii) achieve the optimal allocation?

Problem 4: international economics [20 points]

Consider a small open economy with the following internal demand and supply for the single good:

$$D(P) = 2c - P, \text{ where } c > 0 \text{ is a constant and } S(P) = P.$$

The world price for the good is stable at a relatively low level $P^W < c$.

a) [2 pts] Describe the closed economy equilibrium (consumption, production, market price, import, export, total welfare).

b) [3 pts] How does the market equilibrium change upon opening to free trade? How does welfare change? Describe the open economy equilibrium (consumption, production, market price, import, export, total welfare). Derive welfare as a function of a difference between P^W and autarky price. Illustrate welfare changes graphically using the price-quantity diagram. Depict the area measuring the gains from trade.

c) [4 pts] The government plans to impose an import tariff T (in money per each imported unit of good). What level of the tariff maximizes total welfare in this market? Determine the optimal import tariff as a function of P^W and c. Interpret your findings.

d) [5 pts] Assume that the government cares more about producer surplus than consumer surplus so that producer surplus enters the total social welfare with a multiplier a where $a \in (1,3)$. If the government wants to impose an import tariff, what level of the tariff will be optimal according to the government value function? Determine the optimal import tariff for this situation as a function of a, P^W and c. Interpret your findings.

e) [3 pts] Assume that in the framework of case c) import is harmful to the economy, and the harm in terms of welfare amounts to $ECO = IM^2$ where IM is the quantity imported (e.g. ecological harm from transportation). If the government plans to impose an import tariff, what level would maximize total welfare? Determine the optimal import tariff as a function of P^W and c. Interpret your findings.

f) [3 pts] Assume that in the framework of case e) the government uses quota instead of tariff for import restrictions (it is forbidden to import more than Q units of good in total). What level of quota maximizes total welfare? Is the distribution of welfare benefits different from the case e) if the government sells quota permits on the competitive auction?

Problem 5: microeconomics [20 points]

Suppose that a community with 4 inhabitants has to decide upon the level of a public good B. (For example, B is the amount of free beer for the guests of the traditional community's beer festival. The hosts of the festival are proud when their guests drink the beer and enjoy the festival.) Each inhabitant i makes some contribution $b_i \geq 0$ to the public good. The total quantity of public good provided is $B = b_1 + b_2 + b_3 + b_4$. It costs $10\,b_i$ units of grain to make contribution b_i. Initially each inhabitant is endowed with 50 units of grain. The preferences of the inhabitants are described by the utility functions

$$u_1(B, g_1) = 20\ln(B+1) + g_1, \qquad u_2(B, g_2) = 40\ln(B+1) + g_2,$$
$$u_3(B, g_3) = 30\ln(B+1) + g_3, \qquad u_4(B, g_4) = 10\ln(B+1) + g_4,$$

where B is the level of the public good and $g_i \geq 0$ is the amount of grain consumed by inhabitant i.

a) [3 pts] Suppose that the inhabitants agreed to make equal contributions to the public good. Consider the first inhabitant and find the level of public good which is optimal from his/her point of view.

b) [4 pts] Interpret the choice situation for the first inhabitant as a standard problem of maximizing consumer's utility under budget constraint and illustrate this problem and its solution using a diagram in (B, g1) coordinates (don't forget to label axes).

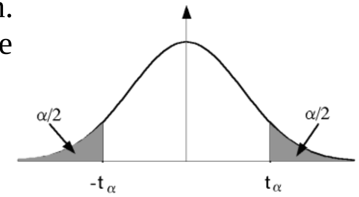
c) [4 pts] Assume that the social welfare measure is the sum of the individual utilities (or, alternatively, the total benefit less the total cost). Find the socially optimal level of the public good B*. Additionally write down the conditions on the levels of grain consumption gi* at the Pareto boundary assuming that grain is transferable between inhabitants.

d) [4 pts] Suppose now that (unlike questions a and b) public good in the community is provided through voluntary contributions. If the total contribution of inhabitants 2, 3 and 4 is B−1 = b2 + b3 + b4, then what is the best response of the first inhabitant b1(B−1)? For which levels of B−1 the first inhabitant prefers to be a "free rider" making zero contribution?

e) [5 pts] Find a pure strategy Nash Equilibrium of a simultaneous voluntary contributions game (each inhabitant i maximizes ui holding the contributions of others fixed). What are the equilibrium contributions $\bar{b}_i$ and the level of the public good $\bar{B}$? (You are not asked to prove the uniqueness of the equilibrium. Just find it and check that it is indeed an equilibrium.) Are there any "free riders" in the equilibrium?

Appendix 1: t-distribution table

The t-distribution table values are critical values of the t distribution. The column headers are the t distribution probabilities (alpha). The row names are the degrees of freedom (df).
Example: with df = 10, for t=2.228, the probability is alpha=0.05



df \ *alpha*	0.1	0.05	0.02	0.01	0.005	0.001
1	6.314	12.71	31.82	63.66	127.3	636.6
2	2.920	4.303	6.965	9.925	14.09	31.60
3	2.353	3.182	4.541	5.841	7.453	12.92
4	2.132	2.776	3.747	4.604	5.598	8.610
5	2.015	2.571	3.365	4.032	4.773	6.869
6	1.943	2.447	3.143	3.707	4.317	5.959
7	1.895	2.365	2.998	3.499	4.029	5.408
8	1.860	2.306	2.896	3.355	3.833	5.041
9	1.833	2.262	2.821	3.250	3.690	4.781
10	1.812	2.228	2.764	3.169	3.581	4.587
11	1.796	2.201	2.718	3.106	3.497	4.437
12	1.782	2.179	2.681	3.055	3.428	4.318
13	1.771	2.160	2.650	3.012	3.372	4.221
14	1.761	2.145	2.624	2.977	3.326	4.140
15	1.753	2.131	2.602	2.947	3.286	4.073
16	1.746	2.120	2.583	2.921	3.252	4.015
17	1.740	2.110	2.567	2.898	3.222	3.965
18	1.734	2.101	2.552	2.878	3.197	3.922
19	1.729	2.093	2.539	2.861	3.174	3.883
20	1.725	2.086	2.528	2.845	3.153	3.850
21	1.721	2.080	2.518	2.831	3.135	3.819
22	1.717	2.074	2.508	2.819	3.119	3.792
23	1.714	2.069	2.500	2.807	3.104	3.768
24	1.711	2.064	2.492	2.797	3.091	3.745
25	1.708	2.060	2.485	2.787	3.078	3.725
26	1.706	2.056	2.479	2.779	3.067	3.707
27	1.703	2.052	2.473	2.771	3.057	3.690
28	1.701	2.048	2.467	2.763	3.047	3.674
29	1.699	2.045	2.462	2.756	3.038	3.659
30	1.697	2.042	2.457	2.750	3.030	3.646
31	1.696	2.040	2.453	2.744	3.022	3.633
32	1.694	2.037	2.449	2.738	3.015	3.622
33	1.692	2.035	2.445	2.733	3.008	3.611
34	1.691	2.032	2.441	2.728	3.002	3.601
35	1.690	2.030	2.438	2.724	2.996	3.591
36	1.688	2.028	2.434	2.719	2.990	3.582
37	1.687	2.026	2.431	2.715	2.985	3.574
38	1.686	2.024	2.429	2.712	2.980	3.566
39	1.685	2.023	2.426	2.708	2.976	3.558
40	1.684	2.021	2.423	2.704	2.971	3.551
41	1.683	2.020	2.421	2.701	2.967	3.544
42	1.682	2.018	2.418	2.698	2.963	3.538
43	1.681	2.017	2.416	2.695	2.959	3.532
44	1.680	2.015	2.414	2.692	2.956	3.526
45	1.679	2.014	2.412	2.690	2.952	3.520
46	1.679	2.013	2.410	2.687	2.949	3.515
47	1.678	2.012	2.408	2.685	2.946	3.510
48	1.677	2.011	2.407	2.682	2.943	3.505
49	1.677	2.010	2.405	2.680	2.940	3.500
50	1.676	2.009	2.403	2.678	2.937	3.496

Appendix 2: F-distribution table

Table of F-statistics for P-value = 0.01
Example: with df1 = 5, df2 = 3 for P-value = 0.01 critical value F(df1, df2) = 28.24

df2\df1	1	2	3	4	5	6	7	8	9	10	20	25	30	35	40	45	50
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.45	99.46	99.47	99.47	99.47	99.48	99.48
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	26.69	26.58	26.50	26.45	26.41	26.38	26.35
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.02	13.91	13.84	13.79	13.75	13.71	13.69
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.55	9.45	9.38	9.33	9.29	9.26	9.24
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.40	7.30	7.23	7.18	7.14	7.11	7.09
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.16	6.06	5.99	5.94	5.91	5.88	5.86
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.36	5.26	5.20	5.15	5.12	5.09	5.07
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.81	4.71	4.65	4.60	4.57	4.54	4.52
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.41	4.31	4.25	4.20	4.17	4.14	4.12
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.10	4.01	3.94	3.89	3.86	3.83	3.81
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	3.86	3.76	3.70	3.65	3.62	3.59	3.57
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.66	3.57	3.51	3.46	3.43	3.40	3.38
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.51	3.41	3.35	3.30	3.27	3.24	3.22
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.37	3.28	3.21	3.17	3.13	3.10	3.08
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.26	3.16	3.10	3.05	3.02	2.99	2.97
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.16	3.07	3.00	2.96	2.92	2.89	2.87
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.08	2.98	2.92	2.87	2.84	2.81	2.78
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.00	2.91	2.84	2.80	2.76	2.73	2.71
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	2.94	2.84	2.78	2.73	2.69	2.67	2.64
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.70	2.60	2.54	2.49	2.45	2.42	2.40
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.55	2.45	2.39	2.34	2.30	2.27	2.25
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.44	2.35	2.28	2.23	2.19	2.16	2.14
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.37	2.27	2.20	2.15	2.11	2.08	2.06
45	7.23	5.11	4.25	3.77	3.45	3.23	3.07	2.94	2.83	2.74	2.31	2.21	2.14	2.09	2.05	2.02	2.00
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.27	2.17	2.10	2.05	2.01	1.97	1.95