ICEF 2020 Olympiad – Solutions

General grading guidelines

There are 5 original tasks created by an international academic committee especially for the purpose of this Olympiad. Each task has been created by a specialist in the area, edited by the academic director of the Olympiad and then blindly reviewed by another specialist independently. These are open ended tasks. The answers will be marked blindly at the discretion of the grading team in a manner uniform for all candidates.

Guidelines

1. Full credit should be reserved for answers that demonstrate mastering the material and the explanation is sufficient and flawless at the discretion of the grader.
2. Partial credit may be given to insufficient answers towards the correct direction at the discretion of the grader.
3. Minimal credit may be given to answers not towards the correct direction, which however indicate that the candidate has some limited knowledge on the subject at the discretion of the grader.
4. Zero credit should be given to answers that are wrong, irrelevant, make no sense, or have random correct elements, at the discretion of the grader.

Any indication of failure to master the material should result to a reduction of credit. For example, if a candidate answers the question perfectly and then attaches an additional part which is irrelevant or wrong, the grader may remove points because the redundancy indicates failure in understanding.

Appeals

Candidates have the right to appeal to the decisions made by the graders according to the HSE Olympiad regulations.

The academic director of the Olympiad will decide if an appeal will be forwarded to the members of the committee for re-grading or it will be terminated as lacking merit. Appeals will be forwarded for re-grading only when one of the following occurs:

1. <u>The candidate sufficiently justifies</u> that the grader has made a specific mistake grading a specific part of the answer.

2. The candidate sufficiently proves that there is a misspecification in some part of the question that allows the candidate's answer to be interpreted as a partially or fully correct response.

3. The candidate sufficiently proves that there is a mistake in a specific part of the question such that there is no correct response.

4. The candidate sufficiently demonstrates that the given answer is also (partially) correct, even though it is not included in this answer key.

Appeals that are insufficiently justified; or do not fall under one of the above 4 cases; or are not in English; or are unclear; or are just asking for higher grades, will be terminated by the academic director of the Olympiad with the indication: "The appeal has no merit (# reason #)".

If the appeal is judged as reasonable by the academic director, a committee member will be asked to re-grade the question. In this case the candidate is possible to receive a higher or a lower or the same grade in comparison to that received in the first degree.

Problem 1: asymmetric information

a) (6 points) If different types issue different shares, the market recognizes each type (separating equilibrium) and assigns fair value to the equity of each type. If they issue the same share, the market assigns the average value to the equity. Hence:

   i. (3 points) No such equilibrium. The market would never finance the bad type, because the bad type's verifiable cash flow, 60, is below the amount of the funds that need to be raised, 100. That is, even if the bad type sells 100% of equity, the investors will value it at $60<100 \Rightarrow$ they will refuse provide financing.

   ii. (3 points) No such equilibrium. The market assigns value (1/2)*120+(1/2)*60=90 to the expected cash flow. As in (i), this is below the amount of the funds that need to be raised $\Rightarrow$ they will refuse provide financing even for 100% of equity.

b) (8 points) Now the minimum size of the required outside financing is 50. Notice also that the entrepreneur's outside option (no financing) is 50, i.e., his cash holdings.

   i. (4 points) No such equilibrium. Now the market would finance the bad type: he would need to sell 5/6 of equity in order for the investors to break even (as (5/6)*60=50). Then, his payoff would be (1/6)*60+60.
   Note: In principle, he could sell more and pocket the excess funds, but that would yield him the same payoff. E.g., if he sells 100%, he will have zero equity share but he will pocket 10 of excess funds and still get his private benefit of 60. This extra reasoning is not required from students.

   However, he would want to deviate and mimic the good type. In the considered separating equilibrium (if exactly *I-A* is raised) the good type would sell 50/120. Hence, by mimicking the good type, the bad type would get (70/120)*60+60>(1/6)*60+60.
   *Note: if the good type raises more than I-A by selling more than 50/120, the bad type would benefit from mimicking even more, as he would have a chance to sell even more overvalued equity. For example, assume the good type sells 100% of equity. Then he raises 120 and pockets 70 of excess money. The bad type, by mimicking the good one, would pocket this 70 plus he would get his private benefit of 60, which is overall even better than (70/120)*60+60. This extra reasoning is not required from students.*

   ii. (4 points) Yes, there is. The market assigns value (1/2)*120+(1/2)*60=90 to the expected cash flow. The minimum share to be sold is then 50/90.
   Then, the good type gets (40/90)*120=160/3>50.
   The bad type receives (40/90)*60+60>50.
   Hence, each type prefers raising funds to doing nothing and keeping 50.
   A deviation to a non-equilibrium offer is not profitable for either type due to the assumption of bad out-of-equilibrium beliefs. Given such beliefs, the minimum out-of-equilibrium share that needs to be offered is 5/6 (from (b.i)). Then, the good type's and the bad type's payoffs would be (1/6)*120 and (1/6)*60+60 which are lower than the respective equilibrium payoffs found above.
   Note 1: If a student does not check "out-of-equilibrium" offers at all, 1 point needs to be subtracted. Checking the minimum out-of-equilibrium offer is enough. (In principle, the entrepreneur could sell more than 5/6 "out-of-equilibrium" and

pocket the excess funds, but that would not benefit either type either. This extra reasoning is not required from students)

Note 2: It is important that we compare the payoffs to 50. It is not enough that both types are able to raise funds, we also need to check that they gain compared to no financing. If a student fails comparing to 50, only 2 points should be given for (ii) (provided that the rest of the solution is correct).

c) (3.5 points) By issuing debt with face value 50, the good type receives the payoff of 120-50=70. This is his full information optimum, he cannot do better than that.

If the bad type also issues debt, he receives 60-50+60=70, which is also his full information solution. He cannot receive more by issuing equity, as (assuming bad out-of-equilibrium beliefs) he we be taken for a bad type and would receive (1/6)*60+60=70 – the same payoff.

There cannot be an equilibrium in which the good type issues equity, as the bad type would immediately mimic him and sell overvalued equity.

Hence, the answer is:

- The good type issues debt
- The bad type issues debt or equity or, actually, any combination of the two.

Note: In this part, considering "out-of-equilibrium" offers is not conceptually fundamental. Hence, students should only be marginally penalized (by subtracting 0.5 points) for not mentioning why, in an equilibrium in which both types issue debt, the bad type would not gain by deviating to selling equity.

d) (2.5 points) Here, the solution is essentially "reversed" compared to (d). The thing is that now, if only outside equity is issued, it is always fairly priced, that is, its value does not depend on the belief about the type, because the expected cash flow is the same for the two types. In contrast, the debt value (for a given face value below 170) does depend on the type, because debt will inevitably be risky in the bad firm (as the face value needs to be at least 100, which exceeds 70).

Formally, the solution is as follows. In the considered equilibrium:

- The good type sells share 100/120 and receives (20/120)*120=20.
- The bad type issues debt with face value 130 (or higher). 130 is the minimum face value the bad type needs to offer to the investors, as in the bad firm they receive (due to the limited liability of the entrepreneur) (1/2)*70+(1/2)*F, where F is the face value, and this must be equal at least 100. The bad type's payoff (due to limited liability) is (1/2)*0+(1/2)*(170-130)=20.

No one has an incentive to deviate: By switching to equity financing, the bad type gets the same 20, as his equity will be fairly priced. Other (out-of-equilibrium) debt offers cannot benefit him due to the assumed bad "out-of-equilibrium" beliefs. By switching to debt financing, the *maximum* the good type can get is his full information payoff of 20 – this is if he believed to be good, he obviously gets even less if considered bad. Other (out-of-equilibrium) equity offers clearly cannot benefit him regardless of the beliefs, as equity is always fairly priced.

Note: In this part, not considering "out-of-equilibrium" offers should not be penalized. Just showing that mimicking each other is not profitable for either type is enough.

Problem 2: econometrics

a. Interpretation: on Lectures: an extra 1% of lectures attended is associated on average with 0.151 increase in the exam grade (keeping the other regressors fixed); on *CS:* an extra 1% of class sections attended is associated on average with 0.319 increase in the exam grade (keeping the other regressors fixed); on G: a student from group G1 has the exam grade 26.103 higher on average than a student from group G2 (keeping the other regressors fixed).

1 point for the interpretation of both coefficients on Lectures and CS; 0 points if even one of them is wrong or missing. 1 point for the interpretation of the coefficient on G. 0 points for not writing something like 'keeping the other regressors fixed', 'all else being equal', etc., and for providing a causal interpretation (i.e., 'an extra 1% of lectures attended leads to 0.151 increase in the test score').

b. $H_0: a_1 = a_2 = 0$ vs $H_A: a_1 \neq 0$ *or* $a_2 \neq 0$ *(0.5 points)* The test statistic is equal to F = $\frac{R^2}{(1-R^2)} \frac{N-2-1}{2} = \frac{0.55}{0.45} \cdot \frac{45}{2} = 27.5$ *(0.5 points)*. It has the F-distribution with (2,45) degrees of freedom under H_0. The critical value for 1% significance level is around 5.11.*(0.5 points)* So we reject the null that the coefficients on all the factors are equal to 0 at 1% significance level (or at 99% confidence level).*(0.5 points)*

Accept also the F-statistic calculation that uses RSS.

c. From model A, since $R_A^2 = 1 - \frac{RSS_A}{TSS}$, $TSS = \frac{RSS_A}{1-R_A^2} = \frac{9394}{1-0.55} \approx 20876$. Then $R_B^2 = 1 - \frac{RSS_B}{TSS} = 1 - \frac{2223}{20876} \approx 0.8935$, and $R_C^2 = 1 - \frac{RSS_C}{TSS} = 1 - \frac{1979}{20877} \approx 0.9052$. $R_C^2 > R_B^2$, but it does not mean that Model C is better, because R^2 cannot decrease when a new variable is added to the regression. We need to compute adjusted R^2 for Models B and C: $adjR_C^2 = 1 - (1 - R_C^2)\frac{N-1}{N-5} \approx 0.896$, $adjR_B^2 = 1 - (1 - R_B^2)\frac{N-1}{N-4} \approx 0.886$. According to the adjusted R^2, the Model C is (a little bit) better.

1 point for calculating TSS, 1 point for calculating both R_A^2 and R_B^2. 2 points for adjusted R^2 calculations and conclusion that model C is better (F-test, AIC, BIC are also appropriate instead of adjusted R^2).

1 point (out of 2) for the last (adjusted R^2 or other appropriate criteria calculation), if it was stated that R^2 is a wrong measure for comparing models with different number of regressors but do not suggest an alternative to it. Also, accept answers that were rounded (correctly) to the nearest tenth or hundredth.

d. To calculate the effect of the additional time spent on the course, we need to transform extra hours to extra percentage of lectures or class sections attended. An extra 1.5-hour lecture attended is equivalent to a 1.5/30=5% increase in lectures attended, and this on average is associated with 5*0.184=0.92 points increase in the exam grade. An extra 1.5-hours class section is equivalent to a 1.5/48=3.125% increase in class sections attended.

For a student from group G2, this on average is associated with 3.125*0.218=0.681 increase in the exam grade. Since 0.92>0.681, it is better to attend an extra lecture.

For a student from group G1, 3.125% increase in class sections attended is associated on average with 3.125*(0.218+0.177)=1.234 increase in the exam grade. So, for a G1 student class sections are preferable to lectures.

1 point for correct conversion of an extra 1.5 hours spent on lectures and classes to the percentage increase in lectures and classes attended; 0.5 points for correct computation of an extra lecture effect. 0.5 points for correct computation of the effect of an extra class *on student from G1; 0.5 points for correct computation of the effect of an extra class on student from G2, 0.5 points for a correct conclusion about both students.*

e. Model C has to be used to answer this question. For G1 students ($G_i = 1$), the effect of the class section attendance is $c_2 + c_4$, and for G2 students ($G_i = 0$) it is c_2. So, the null hypothesis to test is $H_0: c_4 = 0$ *(0.5 points)* against $H_1: c_4 \neq 0$ *(0.5 points)*. The test statistic is equal to $t = \frac{\widehat{c_4}}{s.e.(\widehat{c_4})} \approx \frac{0.177}{0.077} \approx 2.3$ *(0.5 points)*. Under the null, the test statistic has the t-distribution with 43 degrees of freedom *(0.5 points)*. t-distribution table has the row with df=43, and the number 2.3 lays between 2.017 and 2.416 (in this row), and approximately in the same proportion p-value stands between 0.05 and 0.02. So, we calculate $p_{value} \approx 0.05 - (0.05 - 0.02)\frac{2.3-2.017}{2.416-2.017} \approx 0.03$ *(1 point)*. Since $0.05 > 0.03 > 0.01$ we reject H_0 at 5% significance level *(0.5 points)*, but we fail to reject the null at 1% significance level *(0.5 points)*.
Last two conclusions about H_0 may be done without calculation of p-value: the test statistic t=2.3 > 2.017 – the critical value for 5% significance level (H_0 reject on 5%), and t=2.3 < 2.416 – the critical value for 1% significance level (fail to reject the null on 1%). In this case give 0 point for p-value, but 0.5 points for each significance level.

f. Let M denote the math background of the students. Since the course is on mathematical methods in economics, math background might be helpful, so it is reasonable to expect that M is correlated with TS. M is not included in Model A, so it is an omitted variable. To understand, whether this leads to an omitted variable bias, we need to understand whether it is correlated with either Lectures or CS variables. One correct answer would be to say that lecture or class attendance is uncorrelated with math background, so there is no omitted variable bias in Model A. Another correct answer would be to say that perhaps, students with math background tend to attend fewer lectures (or classes, or both), since they think they already have enough knowledge from their high school years. This way the omitted variable M is negatively correlated with variable Lectures (or CS, or both), and hence, there is an omitted variable bias in the estimator of the coefficient a_1 (or a_2, or both). Finally, one might argue that students with math background attend lectures (or class sections, or both) more than students without math background, because the course is more interesting for them, and so the omitted variable is positively correlated with Lectures (or CS, or both), and so there is a positive OVB in in the estimator of the coefficient a_1 (or a_2, or both).
2 points for explaining that there is positive correlation between the omitted variable on the math background and the exam grade TS. 1 point for arguing whether the omitted variable is correlated with Lectures and 1 point for whether it is correlated with CS. Then 1 point for correct conclusion about the presence and the sign of the OVB in the estimator of a_1, and 1 point – for a_2. Since almost all answers are possible, students should correctly infer the sign of the OVB from what they claim about correlations between the omitted variable and the variables included in the model.
1 point only for claiming that math background is an omitted variable, so there is an OVB (without checking whether it's correlated with other variables or not).

Problem 3: finance

a) Since the good can only be consumed or invested, and there is no utility associated with time-0 consumption, it is optimal to invest all at time 0, and then decide how much of the $1+\ r_1$ obtained at time 1 to consume, and how much to reinvest for date 2 consumption, which is given by $C_2 = (1+r_1 - C_1)(1+r_2)$. Thus, we solve

$$\max\ C_1 + \frac{1}{1+\rho} E[(1+r_1 - C_1)(1+r_2)|s]$$

subject to $0 \le C_1 \le 1+\ r_1$. **[2 points]**
This is a maximization of an affine function over a closed interval. Hence, if the slope is positive, the maximum is reached at the interval upper bound :$C_1 =\ 1+\ r_1$, which implies $C_2 = 0$, and if the slope is negative, the maximum is reached at the interval lower bound: $C_1 = 0$ and $C_2 = (1+\ r_1)(1+\ r_2)$. Since the slope equals $1 - \frac{1}{1+\rho}\ E[1+r_2|s]$ we obtain the optimal allocation given in the text. **[2 points]**

b) i) If everybody withdraws at time 1, the amount $1+r_D$ is due. But the bank cannot provide more than what its investment technology yields at time 1, namely $1+r_1$. **[1 point]**

ii) If there is no default at time 1, all withdrawing agents get the promised gross return $1+r_D$. Hence, $\widehat{d_1}/(1+r_D)$ agents must have withdrawn, and $1-\widehat{d_1}/(1+r_D)$ remained invested. **[1 point]**

iii) In case of default at time 2, the payoff of the bank's project is shared equally among remaining creditors. From question b), we know that there are $1-\ \widehat{d_1}/(1+r_D)$ such creditors. The amount reinvested at date 1 is $1+r_1-\widehat{d_1}$, and it yields a gross return $1+r_2$. Hence, the date 2 payment to a remaining creditor is $P \equiv \frac{(1+r_1-\ \widehat{d_1})(1+r_2)}{1-\ \widehat{d_1}/(1+r_D)}$. **[1.5 point]**

iv) Default at time 1 means that the bank has exhausted its resources (but still could not repay all promised amounts). Hence, $\widehat{d_1} = 1+r_1$ in that case and $\delta(\widehat{d_1}, s) = 0$: because nothing has been reinvested at date 1, there is no payoff at date 2. **[1 point]** We can now assume no default at time 1. If there is no default at time 2, the gross return is the promise $1+r_D$. If there is default at time 2, a remaining creditor has reinvested an amount $1+r_D$ (that was on her account at time 1) to obtain the payment P derived in question c). Hence the gross return is $\frac{P}{1+r_D} = \frac{(1+r_1-\ \widehat{d_1})(1+r_2)}{1+r_D-\ \widehat{d_1}}$. **[1.5 point]** The expression of $\delta(\widehat{d_1}, s)$ follows immediately.

c) i) Let $g(d) = \frac{1+r_1-d}{1+r_D-d}$. We have $g'(d) = \frac{r_1-r_D}{(1+r_D-d)^2}$. Hence g is increasing when $r_1 > r_D$. For a given realization of r_2, $\min\big((1+r_2)g, 1+r_D\big)$ is therefore weakly increasing. $\delta(.,s)$ is then the expectation of a function which is state-by-state weakly increasing and therefore weakly increasing too. When $r_1 < r_D$, $g' < 0$ and we conclude similarly that $\delta(.,s)$ is weakly decreasing. **[2.5 points]**

ii) When $r_1 < r_D$, decisions to withdraw are strategic complements: an agent is worse off remaining invested when more other agents decide to leave. When $r_1 > r_D$, decisions to withdraw are strategic substitutes: an agent is better off remaining invested when more other agents decide to leave. **[1.5 point]**

d) i) The only thing to verify is that if everybody else withdraws, then an individual's best response is to also withdraw. When everybody withdraws, there is no good left at time 1, because $r_1 < r_D$. Hence withdrawing is a best response, because staying brings 0 payoff. **[1.5 point]**

ii) We need to find when staying is a best response to everybody staying, i.e. to $\widehat{d_1} = 0$. Given an individual's account at date 1 (after payment of the time 1 interest), withdrawing has a gross return of 1, and staying has an expected gross return $\delta(0, s)$, to be discounted by $1 + \rho$. Hence, everybody staying is an equilibrium when $\frac{\delta(0,s)}{1+\rho} \geq 1$. Given the assumptions on the function f, we see that $\delta(0, .)$ is weakly increasing with $\delta(0,1) = 1 + r_D > 1 + \rho$. Hence, for s sufficiently large, $\frac{\delta(0,s)}{1+\rho} \geq 1$ holds true. **[3 points]**

iii) The equilibrium where everybody withdraws achieves the optimal allocation when $1 + \rho > E[1 + r_2|s]$ but not when $1 + \rho < E[1 + r_2|s]$. When it exists, the equilibrium where nobody withdraws achieves the optimal allocation only if $r_D \geq \sup r_2$, so that depositors always share the full payoff at date 2, as prescribed by the allocation derived in question a. **[1.5 point]**

Problem 4: international economics

a) $Equilibrium\ condition: S(P) = D(P).$

$P^A = c$

$Consumption = D(P^A) = c$

$Production = S(P^A) = c$

$Import = 0$

$Export = 0$

$Welfare = CS + PS = c^2/2 + c^2/2 = c^2$

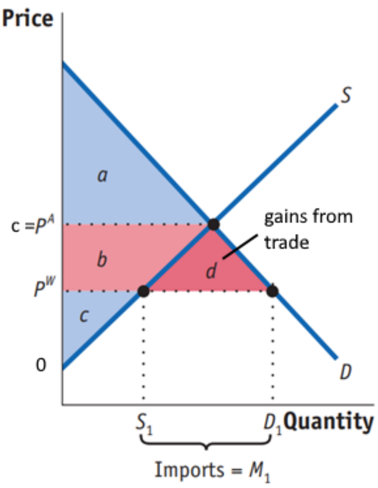
b) Low world price means that right after the trade opens it is beneficial to import the cheap good and not buy more expensive one from the local producers. Consequently the equilibrium price inside the country settles on the level of world price (*P=Pw*), lower than former autarky price. Local producers supply according to their supply function *S(P)*, and the rest of the demand is covered by imports.

$P^* = P^W$

$Consumption = D(P^*) = 2c - P^W$

$Production = S(P^*) = P^W$

$Import = D(P^*) - S(P^*) = (2c - P^W) - P^W = 2c - 2P^W$



$Export = 0$

$Price\ change = \Delta = P^W - c$

$$Welfare = CS + PS = \frac{(2c - P^W)^2}{2} + \frac{(P^W)^2}{2}$$

$$= c^2 + (P^W - c)^2 = c^2 + \Delta^2$$

Welfare increases with free trade compared to autarky, the higher the farther P^W is from the autarky level (the higher is the absolute value of Δ).

Graphically the autarky welfare W_a = *CS* + *PS* = *(a)* + *(b+c)*, while the free trade welfare is *W* = *CS* + *PS* = *(a+b+d)* + *(c)*. Hence, gains from trade are positive and equal to *d*.

c) $Case\ \mathbf{1}: T > c - P^W$ is the forbidding level of the tariff. The local price settles on the autarky level, no trade happens and equilibrium is equivalent to the autarky one. Thus, $Welfare = CS + PS = c^2/2 + c^2/2 = c^2$. Note that the government collects no revenue here, as imports are zero.

$Case\ \mathbf{2}: T \leq c - P^W, then$

$P^* = P^W + T$

$Consumption = D(P^*) = 2c - P^W - T$

$Production = S(P^*) = P^W + T$

$Import = D(P^*) - S(P^*) = (2c - P^W - T) - P^W - T = 2c - 2(P^W + T)$

$Export = 0$

$Government\ revenue = T(2c - 2(P^W + T)) = 2T(c - (P^W + T))$

$$Welfare = CS + PS + GR = \frac{(2c - P^W - T)^2}{2} + \frac{(P^W + T)^2}{2} + 2T\big(c - (P^W + T)\big)$$

$$= c^2 + (P^W + T - c)^2 - 2T(P^W + T - c) = c^2 + (P^W - c)^2 - T^2$$

$$\frac{dWefare}{dT} = -2T < 0$$

Welfare is a decreasing function of the tariff, down until it hits the autarky level. Hence $T^* = 0$ maximizes the welfare in the open economy. Any positive tariff will generate deadweight losses and decrease total welfare as a result.

d) $Case\ \mathbf{1}: T > c - P^W$ is the forbidding level of the tariff. The local price settles on the autarky level, no trade happens and equilibrium is equivalent to the autarky one. Thus, $Welfare = CS + aPS = \frac{c^2}{2} + \frac{ac^2}{2} = c^2(1 + a/2)$. Note that the government collects no revenue here, as imports are zero.

$Case\ 2: T \leq c - P^W, then$

$P^* = P^W + T$

$Consumption = D(P^*) = 2c - P^W - T$

$Production = S(P^*) = P^W + T$

$Import = D(P^*) - S(P^*) = (2c - P^W - T) - P^W - T = 2c - 2(P^W + T)$

$Export = 0$

$Government\ revenue = T(2c - 2(P^W + T)) = 2T(c - (P^W + T))$

$$Welfare = CS + aPS + GR = \frac{(2c - P^W - T)^2}{2} + a\frac{(P^W + T)^2}{2} + 2T\big(c - (P^W + T)\big)$$

$\frac{dWefare}{dT} = -2c + P^W + T + aP^W + aT + 2c - 2P^W - 2T - 2T = 0, SOC = a - 3 < 0$

$T = \frac{P^W(a-1)}{3-a}\ if\ T \leq c - P^W\ holds\ true.$

Note that welfare function is continuous in T, thus we get the following solution

$$T^* = \begin{cases} \dfrac{P^W(a-1)}{3-a}\ if\ a \leq 3 - \dfrac{2P^W}{c} \\ (c - P^W, \infty)\ if\ a > 3 - \dfrac{2P^W}{c} \end{cases}$$

Tariff becomes positive here compared to c) because it is more beneficial for the gov-t to protect the producers.

e) $Case\ \mathbf{1}: T > c - P^W$, then no trade happens and equilibrium is equivalent to the autarky one.

Thus $Welfare = CS + PS = c^2/2 + c^2/2 = c^2$.

Case 2: $T \leq c - P^W$, then welfare function is the same as in the case c) plus ECO = $(2c - 2(P^W + T))$^2 $= 4(P^W + T - c)$^2

$$Welfare = CS + PS + GR - ECO = c^2 + (P^W - c)^2 - T^2 - 4(P^W + T - c)^{\wedge}2$$

$\frac{dWefare}{dT} = -2T - 8(P^W + T - c) = 0, SOC = -10 < 0$

$T^* = 0.8(c - P^W) < c - P^W$, thus, it is always interior solution

f) Same equilibrium as in e) can be achieved by stating quota Q equal to the equilibrium level of imports in case e). Market equilibrium and welfare distribution is identical to case e).

$$Q^* = 2(c - P^W - T^*) = 0.4(c - P^W).$$

CS and PS are identical to case e) since determined by the local price.

Gov-t revenue is also identical to case e) since the buyers of the quota on the competitive auction are ready to pay (and eventually pay) the whole overhead. The overhead in turn is the difference between the price of selling the good (P local) and the price of buying the good (Pw). Which is exactly equal to the per-unit tariff in case e). Therefore, the buyers effectively pay tariff rate at the time of buying the quota permit.

Problem 5: microeconomics

a) For each inhabitant $g_i = 50 - 10\,b_i$. In the case of equal contributions we have $b_i = B/4$, $g_i = 50 - 10\,B/4$ and thus

$$u_1 = 20\ln(B+1) + 50 - 10\,B/4.$$

The maximum of u_1 is obtained when

$$du_1/dB = 20/(B+1) - 10/4 = 0.$$

Thus, $B = 7$ is optimal for the first inhabitant in this case.

b) One can rewrite $g_i = 50 - 10\,B/4$ in the "budget constraint" form:

$$5/2\,B + g_i = 50.$$

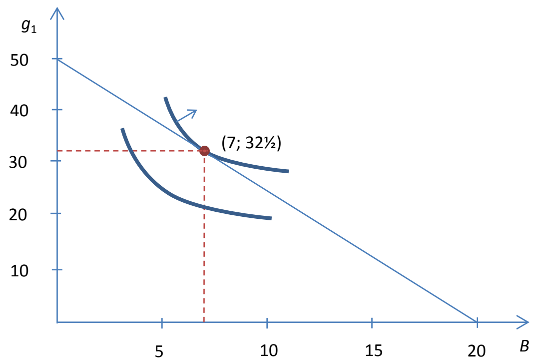
This allows to describe the choice situation of question b as a standard consumer's problem of maximizing utility $u_1 = 20\ln(B+1) + g_1$ under budget constraint

$$\max\ 20\ln(B+1) + g_1,$$
$$5/2\,B + g_1 \le 50.$$

The solution (assuming $g_1 > 0$ and $B > 0$) is found from a system of 2 equations

$$MRS = MU_B/MU_g = 20/(B+1) = P_B/P_g = 5/2,$$
$$5/2\,B + g_i = 50,$$

which gives $B = 7$ and $g_1 = 65/2 = 32½$.



(2 points for the problem and 2 points for the plot).

c) Individual utilities are

$$u_1 = 20\ln(B+1) + 50 - 10\,b_1,\quad u_2 = 40\ln(B+1) + 50 - 10\,b_3,$$
$$u_3 = 30\ln(B+1) + 50 - 10\,b_3,\quad u_4 = 10\ln(B+1) + 50 - 10\,b_4.$$

Defining social welfare W as their sum we obtain

$$W = 100\ln(B+1) + 200 - 10\,B.$$

Alternatively W can be defined as the total benefit less the total cost, that is,

$$W = 100\ln(B+1) - 10\,B.$$

The maximum of W is obtained when

$$dW/dB = 100/(B+1) - 10 = 0,$$

which gives $B^* = 9$.

Another possible solution utilizes the Samuelson's equation

$$MRS_1 + MRS_2 + MRS_3 + MRS_4 = MRTS.$$

That is,

$$20/(B+1) + 40/(B+1) + 30/(B+1) + 10/(B+1) = 10$$

which obviously gives the same answer.

Since $g_i = 50 - 10\,b_i$ for each i and $B = b_1 + b_2 + b_3 + b_4$ the Pareto boundary is described by

$$g_1^* + g_2^* + g_3^* + g_4^* = 200 - 10\,B^* = 110 \qquad (g_i^* \ge 0).$$

Thus the optimum is not unique, but the total consumption of grain should be 110.

(2 points for finding B^* and 2 points for the equation for grain consumption).

d) Knowing that $B = b_1 + B_{-1}$ inhabitant 1 maximizes

$$u_1 = 20\ln(b_1 + B_{-1} + 1) + 50 - 10\,b_1,$$

with respect to b_1 taking B_{-1} as given. If the optimal contribution is positive ($b_1 > 0$), then it is obtained when

$$du_1/db_1 = 20/(b_1 + B_{-1} + 1) - 10 = 0.$$

If the optimal contribution is zero (inhabitant 1 is a free rider), then we must have

$$du_1/db_1 = 20/(0 + B_{-1} + 1) - 10 \leq 0.$$

Thus, there are two cases here and we obtain the following best response:

$$b_1(B_{-1}) = 1 - B_{-1} \text{ for } B_{-1} < 1$$

and

$$b_1(B_{-1}) = 0 \text{ for } B_{-1} \geq 1.$$

(2 points for the internal solution $b_1 = 1 - B_{-1}$ and 2 points for the analysis of the corner solution $b_1 = 0$).

e) One can guess that only inhabitant 2 makes positive contribution in the equilibrium (since he/she has the largest coefficient 40) while other inhabitants are free riders ($\bar{b}_1 = \bar{b}_3 = \bar{b}_4 = 0$).

Knowing that $B = b_2$ inhabitant 2 maximizes

$$u_2 = 40\ln(b_2 + 1) + 50 - 10\,b_2,$$

with respect to b_2. In the optimum

$$du_2/db_2 = 40/(b_2 + 1) - 10 = 0$$

and $\bar{b}_2 = 3$.

Now we check that this is indeed an equilibrium. For example, inhabitant 1 would assume that $B = b_1 + 3$ and thus

$$u_1 = 20\ln(b_1 + 4) + 50 - 10\,b_1.$$

$\bar{b}_1 = 0$ gives the maximum of u_1, since for $b_1 = 0$

$$du_1/db_1 = 20/(b_1 + 4) - 10 = -5 < 0.$$

For inhabitants 3 and 4 the argument is similar.

(3 points for describing the structure of the solution ($\bar{b}_2 > 0$, $\bar{b}_1 = \bar{b}_3 = \bar{b}_4 = 0$) and finding $\bar{b}_2 = 3$; 2 points for checking that inhabitants 1, 3 and 4 would prefer to be free riders).

<u>Uniqueness</u> of pure strategy NE (not required!)

(We assume that $g_i \geq 0$ constraints are not binding). In an equilibrium (A_i is a coefficient before ln)

$$du_i/db_i = A_i/(b_i + B_{-i} + 1) - 10 = A_i/(B + 1) - 10 \leq 0 \text{ or } B \geq A_i/10 - 1$$

for each i. When the inequality is strict, we must have $b_i = 0$ by complementary slackness. This gives $B \geq 3$ for $i = 2$. Hence (by complementary slackness) $i = 1, 3, 4$ are free riders and $B = b_2 = 3$.