Профили:
«Финансовые рынки и финансовые институты»   **КОД – 180**

Ответы и решения

Problem 1 (25 points)
The investor has a wealth of 16 MU (monetary units). She is considering participation in a business project. The project with probability 1/4 will result in gain of 48 MU, and in case of failure the investor will lose 12 MU.

Question 1-1 (5 points). Suppose that the investor is risk averse. What can be said about her decision to participate in the project or not to?

Question 1-2 (10 points). In Questions 1-2 and 1-3 suppose that the investor's preferences are representable by a utility function with the expected utility form (the von Neumann-Morgenstern expected utility function), and that a Bernoulli utility function is $u(x) = \sqrt{x}$.

How does this additional information change your answer in question 1-1? Illustrate the choice of the investor in the wealth-utility space (note on the figure the expected payoff, the value of expected utility in case of participation in the project, the certainty equivalent of the project). Illustrate the choice of the investor in the space of contingent goods (note on the figure the points from which the investor chooses, the typical indifference curves, the slope of the tangent to the indifference curve on the certainty line).

Question 1-3 (10 points). Suppose the investor can consult a consulting company, which could tell her project successfulness with guaranty. However, for the services of a consulting company she needs to pay 7 MU. Will the investor apply to the consulting company? Illustrate the choice of the individual in the space of contingent goods (the figure should be clear to understand from what the investor chooses and what his decision is). Show in the figure the maximum consulting company fee that the investor agrees to pay (it is unnecessary to look for it).

Answer

1. *(5 points)*

The expected payoff is $Ex = \dfrac{1}{4} \cdot 64 + \dfrac{3}{4} \cdot 4 = 19$ **2 points**. As the investor is risk averse the amount of money Ex with certainty better for her than the participation in the project. But if she rejects the project she gets only 16 MU that is less than Ex **1 point**. So there is no sufficient information for the answer **2 points**.

Another way to argue: $Ex = \dfrac{1}{4} \cdot 48 - \dfrac{3}{4} \cdot 12 = 3$ **2 points**. Since the investor is risk averse the amount of money Ex with certainty better for her than the participation in the project. But if she rejects the project she gets 0 that is less than Ex **1 point**. So there is no sufficient information for the answer **2 points**.

2. *(10 points)*

If the investor takes part in the project the expected utility value   is
$$U^{project} = \frac{1}{4}\sqrt{16 + 48} + \frac{3}{4}\sqrt{16 - 12} = 3.5 \quad \textbf{3 points}.$$

If the investor does not participate in the project, then her utility is $u(16) = \sqrt{16} = 4$
1 point.
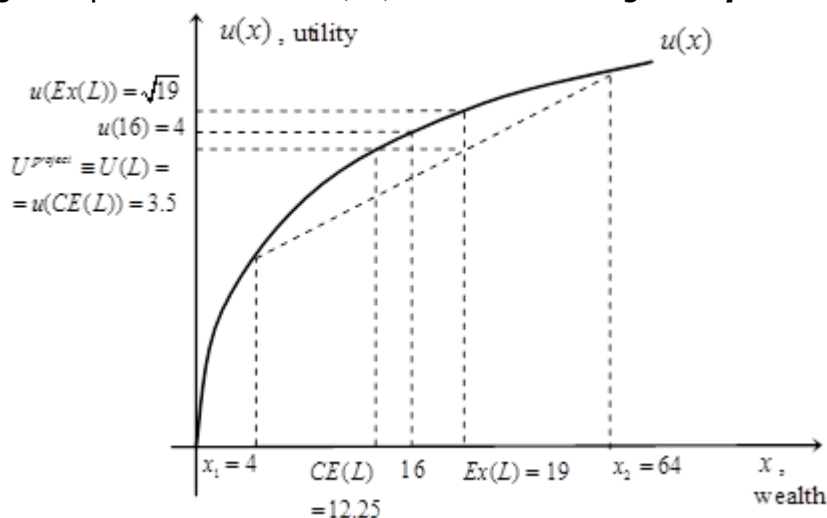Conclusion: the investor rejects the project **1 point**.

Wealth-utility space

The expected payoff: $Ex = \frac{1}{4} \cdot 64 + \frac{3}{4} \cdot 4 = 19$.

By the certainty equivalent definition $u(CE^{project}) = U^{project}$, so $\sqrt{CE^{project}} = 3.5$, and then $CE^{project} = (3.5)^2 = 12.25$ **1 point**.
Right shape of the curve in the figure **1 point**.
Right disposition $CE^{project} = (3.5)^2 = 12.25$ in the figure **1 point**.



Space of contingent goods **2 points**

The investor is a risk averse. This fact follows from the previous figure, which shows that $u(Ex(L)) > U(L)$.

Another way to argue: $u'(x) = \frac{1}{2\sqrt{x}} \underset{x>0}{>} 0$, $u''(x) = -\frac{1}{4x^{3/2}} \underset{x>0}{<} 0$, so $u(x) = \sqrt{x}$ is concave,

and then the investor is a risk averse.
The indifference curves in the space of contingent goods for the risk averse investor are shown the figure below.
If the investor chooses to participate in the project, it means that she chooses ($x_L = 4$, $x_{NL} = 64$) in the space of contingent goods. If the investor chooses not to participate in the project, it means that she chooses ($x_L = 16$, $x_{NL} = 16$). Since the latter is more useful, it lies on a higher indifference curve.
The slope along the certainty line is equal to minus the ratio of the two probabilities so that $MRS_{x_L x_{NL}}(x_L, x_{NL})\big|_{x_L = x_{NL}} = 3$.
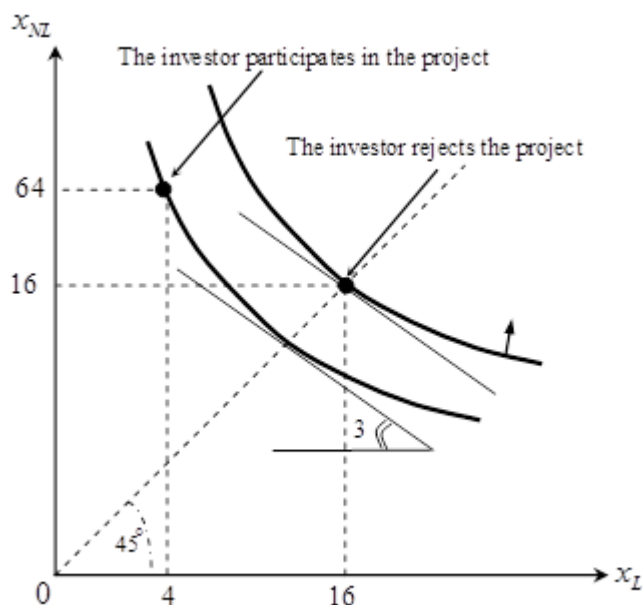
3. (*10 points*)

In the case the consulting firm says that the project is successful, the investor participates in the project. In the case the consulting firm says that the project is failure the investor rejects it. In any case she should pay for consulting company service. So the expected utility is

$U^{consulting} = \frac{1}{4}\sqrt{16+48-7} + \frac{3}{4}\sqrt{16-7} = \frac{1}{4}\sqrt{57} + \frac{3}{4}\sqrt{9} > \frac{1}{4}\cdot 7 + \frac{3}{4}\cdot 3 = 4$ **4 points**. As we

have $U^{consulting} > u(16) = 4$ (**2 points**) the investor contacts a consulting company (**1 point**).

Space of contingent goods **3 points**

q_{max} – the maximum fee of the consulting firm services that the investor is willing to pay. If the services were free, then contracting a consulting firm would mean that the investor chooses ($x_L = 64$, $x_{NL} = 16$) in the contingent goods space. In the figure q_{max} are marked with different length segments on the vertical and horizontal axes due to inconsistent scale.
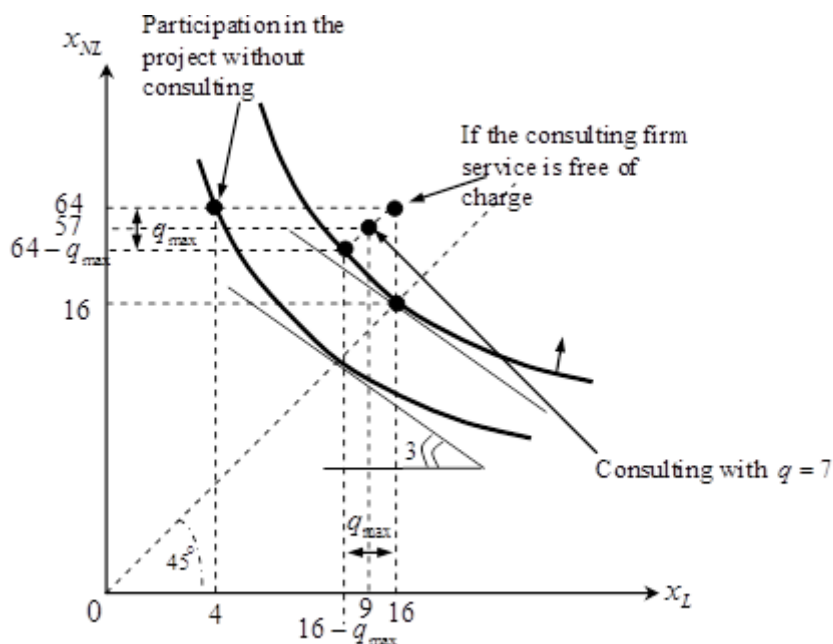
If there is no contingent goods space illustration but right utility function for q_{max}

$$U = \frac{1}{4} \cdot \sqrt{16 + 48 - q_{max}} - \frac{3}{4} \cdot \sqrt{16 - q_{max}} = 4 \; \textbf{\textit{1 point}}.$$

Problem 2 (25 points)

Consider the closed economy in the short run with rigid prices and nominal wages. Price level is equal to 1. The Keynesian consumption function is given by the following expression: $C = 40 + 0.8Yd$, where Yd is disposable income. Income tax rate is equal to 25%. Government spending is exogenous and equal to 40, transfers are absent. Investment function is the following: $I = 140 - 10i$, where i is the nominal interest rate in the economy measured in percentages. Nominal money supply is exogenous and equal to 100. Keynesian money demand function is given by: $m^d = 0.2Y - 5i$.

Question 2-1 (5 points). Find the equilibrium levels of Y, i and the government budget balance.

Question 2-2 (8 points). Assume that the central bank targets nominal interest rate at the equilibrium level found in (1), while due to the exogenous shock autonomous investment increases by 60. Find the corresponding change in the nominal money supply and the new equilibrium level of income. Illustrate these changes on the IS-LM diagram and explain intuitively.

Question 2-3 (8 points). Central bank does not target nominal interest rate, while due to the exogenous shock autonomous investment increases by 60. Find the corresponding change in the income tax rate, if the government wants to keep the equilibrium level of income found in (2-1) unchanged. Illustrate these changes on the IS-LM diagram and explain intuitively. What will happen to the nominal interest rate in equilibrium?

Question 2-4 (4 points). Assume that as in (2-2) central bank targets nominal interest rate and increases nominal money supply to offset the influence of the investment shock. But the government wants to keep the equilibrium level of income found in (2-1) unchanged. How should government spending be changed? What will happen to the nominal interest rate in equilibrium?

Answer:
1. (**5 баллов**)
Выведем уравнения кривых IS и LM:

$$\begin{cases} IS: Y = C + I + G = 40 + 0,8(Y - 0,25Y) + 140 - 10i + 40 \\ LM: \dfrac{M^s}{P} = m^d \Longrightarrow 100 = 0,2Y - 5i \end{cases}$$

Решив данную систему уравнений, получим $Y = 525; i = 1$

Сальдо государственного бюджета $T - G = 0,25Y - 40 = 91,25$

$Y = C + I + G$ – **_1 балл_**
Уравнение кривой IS – **_1 балл_**
Уравнение кривой LM – **_1 балл_**
Равновесные значения Y и i – **_1 балл_**
Сальдо государственного бюджета – **_1 балл_**

2. _(8 баллов)_

После изменений функция инвестиций имеет вид $I = 200 - 10i$; новое уравнение кривой $IS: Y = 700 - 25i$

Центральный банк таргетирует номинальную ставку процента на уровне $i = 1$, отсюда получаем $Y = 675$. Подставив в уравнение LM, получим, что $M^s = 0,2 * 675 - 5 * 1 = 130$, откуда изменение денежной массы равно $130 - 100 = 30$.

На диаграмме IS-LM в координатах $(Y; i)$ кривая IS сдвинется вправо из-за роста инвестиций и кривая LM сдвинется вправо из-за роста предложения денег. Другой допустимый вариант иллюстрации – горизонтальная кривая LM на уровне $i = 1$.

Интуитивное объяснение: с ростом инвестиций растёт выпуск, из-за чего растёт трансакционный спрос на деньги, что приводит к росту ставки процента. Для удержания ставки процента на прежнем уровне центральный банк должен увеличить предложение денег, что позволит снизить цену денег (ставку процента) до изначального уровня.

Новое уравнение IS – **_2 балла_**
Равновесное значение Y - **_1 балл_**
Изменение денежной массы – **_1 балл_**
Графическая иллюстрация – **_2 балла_**
Интуитивное объяснение – **_2 балла_**

3. _(8 баллов)_

Получаем следующую систему уравнений IS-LM:

$$\begin{cases} IS: Y = 280 + 0,8(1 - t)Y - 10i \\ LM: 100 = 0,2Y - 5i \end{cases}$$

где t – ставка подоходного налога.

Поскольку правительство поддерживает выпуск на прежнем уровне, $Y = 525$. Подставив в LM, получим $i = 1$, откуда равновесное значение ставки налога $t = \frac{11}{28}$.

На графике IS-LM кривая LM не изменится, а кривая IS сначала сдвинется вправо из-за роста инвестиций, а затем повернётся вокруг точки пересечения с осью i (станет более крутой) так, что итоговые значения выпуска и ставки процента совпадут с изначальными.

Интуитивное объяснение: с увеличением инвестиций выпуск растёт и для возвращения его значения на исходный уровен правительство должно

увеличить ставку подоходного налога. Рост ставки подоходного налога ведёт к сокращению располагаемого дохода домохозяйств, а значит к сокращению потребления и ВВП.

Система IS-LM – **2 балла**
Значения выпуска и ставки процента – **1 балл**
Ставка налога – **1 балл**
Графическая иллюстрация – **2 балла**
Интуитивное объяснение – **2 балла**

4. (**4 балла**)

Поскольку ЦБ таргетирует ставку процента, в равновесии $i = 1$. Подставив в IS, получим $230 + G = 0{,}4Y$, откуда $Y = 525$ может быть равновесным выпуском только при значении государственных закупок $G = -20$. Отрицательным значение государственных закупок быть не может, поэтому правительство не сможет сохранить выпуск на изначальном уровне $Y = 525$, если ЦБ по-прежнему будет таргетировать ставку процента.

Засчитывалась и иная трактовка условия: если участник олимпиады считал, что ЦБ перестаёт таргетировать ставку процента, то в равновесии при $Y = 525$ значение государственных закупок всё равно получалось отрицательным, то есть и при такой трактовке условия $Y = 525$ достичь нельзя.

Формализация условия (запись IS и LM) – **2 балла**
Нахождение государственных закупок – **1 балл**
Вывод о том, что нельзя достичь значения $Y = 525$ **- 1 балл**