

**Профили:**

«Стратегическое управление финансами фирмы/ The Strategic Corporate Finance

**КОД – 181**

**Ответы и решения**

**Problem 1 (25 points)**

The investor has a wealth of 16 MU (monetary units). She is considering participation in a business project. The project with probability  $1/4$  will result in gain of 48 MU, and in case of failure the investor will lose 12 MU.

**Question 1-1 (5 points).** Suppose that the investor is risk averse. What can be said about her decision to participate in the project or not to?

**Question 1-2 (10 points).** In Questions 1-2 and 1-3 suppose that the investor's preferences are representable by a utility function with the expected utility form (the von Neumann-Morgenstern expected utility function), and that a Bernoulli utility function is  $u(x) = \sqrt{x}$ .

How does this additional information change your answer in question 1-1? Illustrate the choice of the investor in the wealth-utility space (note on the figure the expected payoff, the value of expected utility in case of participation in the project, the certainty equivalent of the project). Illustrate the choice of the investor in the space of contingent goods (note on the figure the points from which the investor chooses, the typical indifference curves, the slope of the tangent to the indifference curve on the certainty line).

**Question 1-3 (10 points).** Suppose the investor can consult a consulting company, which could tell her project successfulness with guaranty. However, for the services of a consulting company she needs to pay 7 MU. Will the investor apply to the consulting company? Illustrate the choice of the individual in the space of contingent goods (the figure should be clear to understand from what the investor chooses and what his decision is). Show in the figure the maximum consulting company fee that the investor agrees to pay (it is unnecessary to look for it).

***Answer***

**1. (5 points)**

The expected payoff is  $Ex = \frac{1}{4} \cdot 64 + \frac{3}{4} \cdot 4 = 19$  **2 points**. As the investor is risk averse the amount of money  $Ex$  with certainty better for her than the participation in the project. But

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if she rejects the project she gets only 16 MU that is less than  $Ex$  **1 point**. So there is no sufficient information for the answer **2 points**.

Another way to argue:  $Ex = \frac{1}{4} \cdot 48 - \frac{3}{4} \cdot 12 = 3$  **2 points**. Since the investor is risk averse the amount of money  $Ex$  with certainty better for her than the participation in the project. But if she rejects the project she gets 0 that is less than  $Ex$  **1 point**. So there is no sufficient information for the answer **2 points**.

**2. (10 points)**

If the investor takes part in the project the expected utility value is

$$U^{project} = \frac{1}{4} \sqrt{16+48} + \frac{3}{4} \sqrt{16-12} = 3.5 \text{ } \mathbf{3 \text{ points}}.$$

If the investor does not participate in the project, then her utility is  $u(16) = \sqrt{16} = 4$  **1 point**.

Conclusion: the investor rejects the project **1 point**.

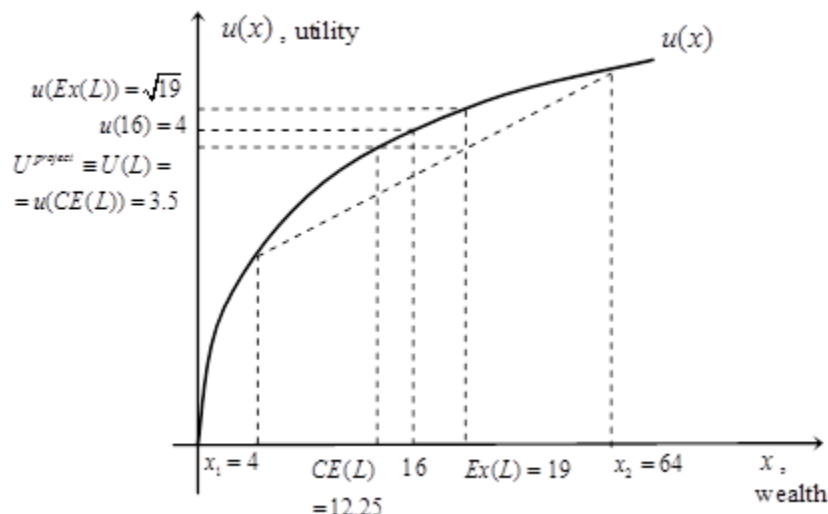
*Wealth-utility space*

The expected payoff:  $Ex = \frac{1}{4} \cdot 64 + \frac{3}{4} \cdot 4 = 19$ .

By the certainty equivalent definition  $u(CE^{project}) = U^{project}$ , so  $\sqrt{CE^{project}} = 3.5$ , and then  $CE^{project} = (3.5)^2 = 12.25$  **1 point**.

Right shape of the curve in the figure **1 point**.

Right disposition  $CE^{project} = (3.5)^2 = 12.25$  in the figure **1 point**.



*Space of contingent goods 2 points*

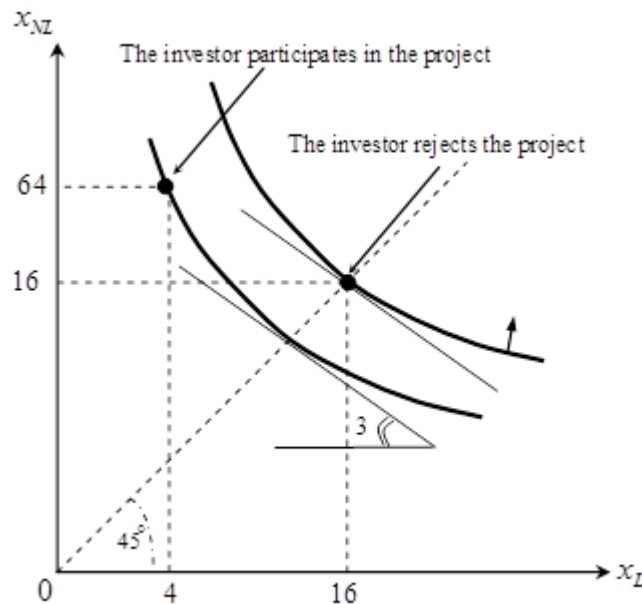
The investor is a risk averse. This fact follows from the previous figure, which shows that  $u(Ex(L)) > U(L)$ .

Another way to argue:  $u'(x) = \frac{1}{2\sqrt{x}} > 0$ ,  $u''(x) = -\frac{1}{4x^{3/2}} < 0$ , so  $u(x) = \sqrt{x}$  is concave, and then the investor is a risk averse.

The indifference curves in the space of contingent goods for the risk averse investor are shown the figure below.

If the investor chooses to participate in the project, it means that she chooses  $(x_L = 4, x_{NL} = 64)$  in the space of contingent goods. If the investor chooses not to participate in the project, it means that she chooses  $(x_L = 16, x_{NL} = 16)$ . Since the latter is more useful, it lies on a higher indifference curve.

The slope along the certainty line is equal to minus the ratio of the two probabilities so that  $MRS_{x_L x_{NL}}(x_L, x_{NL}) \Big|_{x_L = x_{NL}} = 3$ .



**3. (10 points)**

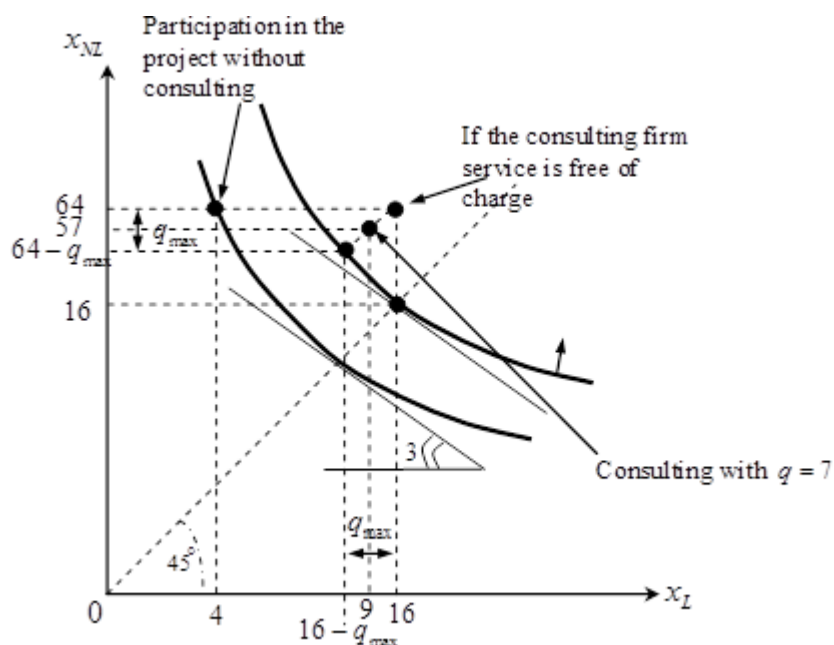
In the case the consulting firm says that the project is successful, the investor participates in the project. In the case the consulting firm says that the project is failure the investor rejects it. In any case she should pay for consulting company service. So the expected utility is

$$U^{consulting} = \frac{1}{4}\sqrt{16+48-7} + \frac{3}{4}\sqrt{16-7} = \frac{1}{4}\sqrt{57} + \frac{3}{4}\sqrt{9} > \frac{1}{4} \cdot 7 + \frac{3}{4} \cdot 3 = 4 \text{ 4 points. As we have}$$

$U^{consulting} > u(16) = 4$  (2 points) the investor contacts a consulting company (1 point).

### Space of contingent goods 3 points

$q_{\max}$  – the maximum fee of the consulting firm services that the investor is willing to pay. If the services were free, then contracting a consulting firm would mean that the investor chooses ( $x_L = 64$ ,  $x_{NL} = 16$ ) in the contingent goods space. In the figure  $q_{\max}$  are marked with different length segments on the vertical and horizontal axes due to inconsistent scale.



If there is no contingent goods space illustration but right utility function for  $q_{\max}$   $U = \frac{1}{4}$ .

$$\sqrt{16 + 48 - q_{\max}} - \frac{3}{4} \cdot \sqrt{16 - q_{\max}} = 4 \text{ I point.}$$

### **Problem 2 (25 points)**

Consider the closed economy in the short run with rigid prices and nominal wages. Price level is equal to 1. The Keynesian consumption function is given by the following expression:  $C = 40 + 0.8Y_d$ , where  $Y_d$  is disposable income. Income tax rate is equal to 25%. Government spending is exogenous and equal to 40, transfers are absent. Investment function is the following:  $I = 140 - 10i$ , where  $i$  is the nominal interest rate in the economy measured in percentages. Nominal money supply is exogenous and equal to 100. Keynesian money demand function is given by:  $m^d = 0.2Y - 5i$ .

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**Question 2-1 (5 points).** Find the equilibrium levels of  $Y$ ,  $i$  and the government budget balance.

**Question 2-2 (8 points).** Assume that the central bank targets nominal interest rate at the equilibrium level found in (1), while due to the exogenous shock autonomous investment increases by 60. Find the corresponding change in the nominal money supply and the new equilibrium level of income. Illustrate these changes on the IS-LM diagram and explain intuitively.

**Question 2-3 (8 points).** Central bank does not target nominal interest rate, while due to the exogenous shock autonomous investment increases by 60. Find the corresponding change in the income tax rate, if the government wants to keep the equilibrium level of income found in (2-1) unchanged. Illustrate these changes on the IS-LM diagram and explain intuitively. What will happen to the nominal interest rate in equilibrium?

**Question 2-4 (4 points).** Assume that as in (2-2) central bank targets nominal interest rate and increases nominal money supply to offset the influence of the investment shock. But the government wants to keep the equilibrium level of income found in (2-1) unchanged. How should government spending be changed? What will happen to the nominal interest rate in equilibrium?

**Answer:**

**1. (5 баллов)**

Выведем уравнения кривых IS и LM:

$$\begin{cases} IS: Y = C + I + G = 40 + 0,8(Y - 0,25Y) + 140 - 10i + 40 \\ LM: \frac{M^s}{P} = m^d \Rightarrow 100 = 0,2Y - 5i \end{cases}$$

Решив данную систему уравнений, получим  $Y = 525; i = 1$

Сальдо государственного бюджета  $T - G = 0,25Y - 40 = 91,25$

$Y = C + I + G$  – **1 балл**

Уравнение кривой IS – **1 балл**

Уравнение кривой LM – **1 балл**

Равновесные значения  $Y$  и  $i$  – **1 балл**

Сальдо государственного бюджета – **1 балл**

**2. (8 баллов)**

После изменений функция инвестиций имеет вид  $I = 200 - 10i$ ; новое уравнение кривой IS:  $Y = 700 - 25i$

Центральный банк таргетирует номинальную ставку процента на уровне  $i = 1$ , отсюда получаем  $Y = 675$ . Подставив в уравнение LM, получим, что

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$M^s = 0,2 * 675 - 5 * 1 = 130$ , откуда изменение денежной массы равно  
 $130 - 100 = 30$ .

На диаграмме IS-LM в координатах  $(Y; i)$  кривая IS сдвинется вправо из-за роста инвестиций и кривая LM сдвинется вправо из-за роста предложения денег. Другой допустимый вариант иллюстрации – горизонтальная кривая LM на уровне  $i = 1$ .

Интуитивное объяснение: с ростом инвестиций растёт выпуск, из-за чего растёт транзакционный спрос на деньги, что приводит к росту ставки процента. Для удержания ставки процента на прежнем уровне центральный банк должен увеличить предложение денег, что позволит снизить цену денег (ставку процента) до изначального уровня.

Новое уравнение IS – **2 балла**

Равновесное значение  $Y$  - **1 балл**

Изменение денежной массы – **1 балл**

Графическая иллюстрация – **2 балла**

Интуитивное объяснение – **2 балла**

### 3. (8 баллов)

Получаем следующую систему уравнений IS-LM:

$$\begin{cases} IS: Y = 280 + 0,8(1 - t)Y - 10i \\ LM: 100 = 0,2Y - 5i \end{cases}$$

где  $t$  – ставка подоходного налога.

Поскольку правительство поддерживает выпуск на прежнем уровне,  $Y = 525$ .

Подставив в LM, получим  $i = 1$ , откуда равновесное значение ставки налога  $t = \frac{11}{28}$ .

На графике IS-LM кривая LM не изменится, а кривая IS сначала сдвинется вправо из-за роста инвестиций, а затем повернётся вокруг точки пересечения с осью  $i$  (станет более крутой) так, что итоговые значения выпуска и ставки процента совпадут с изначальными.

Интуитивное объяснение: с увеличением инвестиций выпуск растёт и для возвращения его значения на исходный уровень правительство должно увеличить ставку подоходного налога. Рост ставки подоходного налога ведёт к сокращению располагаемого дохода домохозяйств, а значит к сокращению потребления и ВВП.

Система IS-LM – **2 балла**

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Значения выпуска и ставки процента – *1 балл*

Ставка налога – *1 балл*

Графическая иллюстрация – *2 балла*

Интуитивное объяснение – *2 балла*

### 4. (4 балла)

Поскольку ЦБ таргетирует ставку процента, в равновесии  $i = 1$ . Подставив в IS, получим  $230 + G = 0,4Y$ , откуда  $Y = 525$  может быть равновесным выпуском только при значении государственных закупок  $G = -20$ . Отрицательным значение государственных закупок быть не может, поэтому правительство не сможет сохранить выпуск на изначальном уровне  $Y = 525$ , если ЦБ по-прежнему будет таргетировать ставку процента.

Засчитывалась и иная трактовка условия: если участник олимпиады считал, что ЦБ перестает таргетировать ставку процента, то в равновесии при  $Y = 525$  значение государственных закупок всё равно получалось отрицательным, то есть и при такой трактовке условия  $Y = 525$  достичь нельзя.

Формализация условия (запись IS и LM) – *2 балла*

Нахождение государственных закупок – *1 балл*

Вывод о том, что нельзя достичь значения  $Y = 525$  - *1 балл*

## **STRATEGIC CORPORATE FINANCE (PROFILE 181)**

Solution comments

### **PART 2. Corporate Finance**

Solve the problems 3 and 4 given below

#### **Problem #3 (25 points)**

You are asked to advise on an investment project. Company BCD plans to implement a project that will be financed with debt and equity. Shareholders have decided to implement the project with help of a separate new entity – company Z. Company Z will be created specifically to realize the project.

You are given the following info regarding the project:

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Project has 3 years maturity. Sales are expected to be 400, 600 and 800 mln. rubles for the corresponding three years. COGS (excluding depreciation expense) will comprise 60% of Sales. Capital expenditures will reflect acquisition of equipment for 270 mln. rubles. It will be fully depreciated using straight-line approach. Net working capital management guidelines require current assets to be at 30% of expected EBITDA in a corresponding year. Current liabilities are planned to be at 10% of expected COGS in a corresponding year. Corporate income tax rate is 20%. Interest rate on debt capital is 10%. Required return on unlevered equity capital is 20%.

Project will be financed with debt and equity capital. Debtholders can provide 120 mln. rubles with 10% required return. The rest will be financed with equity capital.

Managers agreed to a special debt repayment scheme. 120 mln. of debt will be retired in three equal installments of 40 mln. at the end of each of three years. All of three repayments will be financed with equity issues. Such a repayment scheme will result in changing capital structure. After the final debt repayment company Z will realize itself to be financed only with equity.

**Question 3.1 (5 points)** Project Net Income for 3 years. Assume the project is financed with equity.

### *Comments*

Important – answer the question asked.

Full mark was given for three figures of net profit with assumption of no debt used.

**Question 3.2 (10 points)** Project investment cashflows. Assume the project is financed with equity only.

### *Comments*

Important – answer the question asked about investment flows. Overall FCFF table was not asked for.

Full mark was given for One capital expenditure flow and Four figures of changes of Net Working Capital

**Question 3.3 (5 points)** Determine the APV of the project. Should the project be implemented.

### *Comments*

It was required to show correct application of APV approach. No need to present WACC or FCFE approaches

Full mark was given to NPV(basecase) & PVTS calculation

**Question 3.4 (5 points)** Determine the change of equity value at the moment the project finishes after 3 years of implementation. State assumptions.

### *Comments*

When project finishes, company is left with now assets and no debt and the last cash flow will be associated with the last dividend equal to FCFE last period. Equityholders will receive that and will reinvest in financial markets with NPV=0. However value of equity decreases at the end of the last period, shareholders will be indifferent (since they will receive the last dividend cash flow).



**Problem #4 (25 points)**

A company X is all equity financed. Its last year's average equity was \$1500. Accounting rate of return on equity always was and is planned to be 10% forever. Management of the company follows a NO growth policy which leads to 100% payout policy. Management utilizes company's assets in such a way that they completely renew every four years. In other words company assets have 5 years of economic life. Suppose there is no net working capital used in X's business. Capital market analytics say that the required return on X's equity is 16%.

Today is December 31, 2020, and payout together with investment decisions have to be made. Shareholders of the company have just made a hard decision to change the whole management team. New managers received a task to increase the value of X's stock without changing the whole business model. Managers cannot influence company's efficiency. They offer to increase capital spending (follow stable growth model). Managers offer to reinvest 20% of net income. This plan is going to be released to the public on January 1, 2021 and cash will be delivered to shareholders right after that (suppose delivery happens on the same date). There are 100 shares outstanding today.

**Question 4.1 (5 points)** Determine the value of stock under old zero growth policy.

***Comments***

Full mark was given for correct application of dividend discount model with zero growth. One should mention that the first expected dividend payment is not accounted for in a regular Gordon model. Therefore, this one is a correct way

$$P(0) = \text{DPS}(0) + \text{DPS}(1)/R_e$$

Of course dividend payments are equal to each other

**Question 4.2 (10 points)** Determine the value of stock under new stable growth policy.

***Comments***

Full mark was given for correct application of dividend discount model with stable growth. One should mention that the first expected dividend payment is not accounted for in a regular Gordon model. Therefore, this is a correct way

$$P(0) = \text{DPS}(0) + \text{DPS}(1)/(R_e - g)$$

The first dividend is smaller compared to corresponding dividend under zero growth

**Question 4.3 (5 points)** There should be a clear explanation for tradeoff between dividend stream patterns and stock values. Zero growth policy offers greater dividends per share (DPS) at the beginning and no growth; stable growth policy offers smaller DPS from the beginning which grow in the future. Do you think you should always expect a stock value to increase when a higher growth rate (greater capex, or reinvestment rate) is proposed? Explain

*Comments*

Full mark was given for explaining that additional investments in assets with negative NPV will destroy value of our company. Our firm has assets that generate 10% return. Shareholders require 16% as a fair compensation. Every dollar invested will not bring 16% but offer 10%. When growth rate is modelled to increase, it is associated with increase in capital expenditures in assets with  $NPV < 0$ . This is the only acceptable explanation for the fact that stock value in  $q_2$  is smaller than in  $q_1$ .

**Question 4.4 (5 points)** Offer optimal investment policy that will maximize the value of the X company.

*Comments*

Full mark was given for the following proposal : Since  $NPV < 0$  and we can't influence this fact & force it to be positive, there is no reason for our business to exist. Capex should be zero (no reinvestments). In that case assets will depreciate to zero in 5 year and our firm will cease to exist. However,  $Capex = 0$  is the investment strategy that will allow shareholders the quickest way to reinvest in financial markets that always offer  $E(NPV) = 0$ .