

# International Youth Olympiad 2018

## training version

1. Find the minimal value of a function  $f(x) = \frac{5x^2+10x+13}{x^2+2x+3}$ . (20 points)
2. Calculate a sum of all positive integer roots of an equation  $|2x^2+4x+5|-|x-28| = 2x^2+5x-23$  which are divisible by 3. (20 points)
3. Consider a common tangent to parabolas  $y = x^2 + 1$  and  $y = -x^2 - 1$ . Find the absolute value of its slope. (20 points)
4. Many participants of a mathematical class also take part in a class on programming. The share of these people among all the participants of the mathematical class is not less than 95.5% and not more than 96.5%. What is the minimal number of participants of the mathematical class when this situation is possible? (15 points)
5.  $B$  and  $C$  are two points on the same side of an angle with a vertex  $A$ . Let  $X$  be a point on the other side of the angle  $A$ , such that angle  $BXC$  reaches its maximal value (among all possible positions of  $X$ ). Find the length of the segment  $AX$ , provided that angle  $A$  equals to  $60^\circ$  and the lengths of the segments  $AB$  and  $AC$  equal to 7 and 12 respectively. (15 points)
6. Peter wants to paint the stairs and he has three colours of paint to do that: blue, red, and white. He doesn't have to use all the colours (or even two of them) but he still wants to follow some rules: 1) each step of the stairs can be coloured in one colour only; 2) consecutive steps may not be of the same colour; 3) a blue step (if any) must have a red and a white neighbour. How many different colour patterns exist for 12-step stairs? (10 points)