

HSE Global Scholarship Competition – 2020

10th grade, training version

1. (7 points) Three sides of a right triangle form an arithmetic progression. Determine the sin of its least angle.
2. (7 points) Solve the inequality $4x - 5 > \sqrt{1 + x(x + 2)} + \sqrt{-3x^2 + 6x + 24}$.
3. (7 points) Compute the value of an expression $\operatorname{arctg}(\sqrt{2\sqrt{3}} + 1) - \operatorname{arctg}(\sqrt{2\sqrt{3}} - 1)$.
4. (7 points) Determine a sum of all integer a for which both different roots of an equation $x^2 + 4x + a - 4 = 0$ belong to an interval $(-6; 0)$.
5. (7 points) Solve the system:

$$\begin{cases} 3x^2y^2 + x^2 - 3xy = 7 \\ 10x^2y^2 + 3x^2 - 20xy = 3 \end{cases}$$

6. (7 points) Determine the quantity of 4-digit numbers not divisible by 1000 and having both the first and the last digits even.
7. (13 points) Two touching circles on a plane Γ_1 and Γ_2 with radii equal to 4 and 9 touch their mutual external tangent l . Determine the radius of a circle touching Γ_1, Γ_2 and l (the one which is surrounded with segments of the circles and the line l).
8. (13 points) Find the distance between point sets defined on a coordinate plane with equations $4x + 3y - 7 = 0$ and $x^2 + y^2 - 14x - 18y + 121 = 0$.
By distance between point sets we suppose the minimal distance between a point of one set and a point of another set.
9. (16 points) Let's denote by x the quantity of digits in a number 2^{2019} , by y we denote the quantity of digits in a number 5^{2019} . Determine $x + y$.
10. (16 points) There are 11 bus stops on a bus route (counting the first one). 10 passengers got into the bus on the first stop. The total of entering and exiting persons on any other stop besides the the last one is equal to 10 as well. Also somebody noticed that each passenger visited not more then 6 stops (i.e. starting from a stop № M and exiting not later than on a stop № $M + 5$). The bus wasn't empty at any moment of driving. Determine the maximal possible number of passengers taking the bus at the same moment.