

# HSE Global Scholarship Competition – 2020

## 11<sup>th</sup> grade, training version

1. (7 points) Given  $x + y = 2\sqrt{11}$ ,  $xy = -5$  find the distance between  $x$  and  $y$  on the real line.
2. (7 points) Determine a set of all possible  $a$  such that an equation  $|x| + 5a = ax - 1$  has two real roots.
3. (7 points) All points  $x$  of the real line satisfying the following condition were highlighted.

$$\cos^2 \pi x + \frac{\cos 2\pi x}{x^2 - 1} \geq \sin^2 \pi x$$

Find the total length of the intervals of the highlighted points within -100 and 100.

4. (7 points) Find a sum of all roots of an equation  $25^x - e^2 \cdot 5^x = 5^{x+2} - 125$ .
5. (7 points) Find a sum of values of  $a$  for which the following system has just 2 solutions:

$$\begin{cases} |x + y| \geq a + 3 \\ x^2 + y^2 = (a + 2)^2 \end{cases}$$

6. (7 points) The standard of writing a date in the USA is writing the month's number first, then the day's number and finally the year. The standard in Europe supposes the day's number first then the month and then the year. How many days withing a year might be read incorrectly without knowing which standard is used?
7. (13 points) Three unit balls (i.e. with radius equal to 1) touch some plane in a three dimensional space and also touch each other. Determine radius of a ball touching the mentioned above plane and 3 balls.
8. (13 points) A small furniture company produces bookcases and sideboards. Production of a bookcase requires  $4/3$  m<sup>2</sup> of wood board,  $4/3$  kg of nails and  $2/3$  man-hours of working resources. Production of a sideboard requires 2 m<sup>2</sup> of wood board, 1.5 kg of nails and 2 man-hours. Profit for selling a bookcase equals to 5000 rub. and profit for selling a sideboard equals to 12000 rub. The company's monthly resources are 180 m<sup>2</sup> of wood board, 165 kg of nails and 160 man-hours. Determine the maximal possible monthly profit of the company.
9. (16 points) A teacher computed all the remainders of the division of 365 by 1, 2, 3, ..., 365. He computed the sum of the remainders and wrote them on a blackboard. His student computed all the remainders of the division of 366 by 1, 2, 3, ..., 366, summed up the remainders and wrote them on a scratch. Whose number is bigger?
10. (16 points) A square  $n \times n$  was cut into squares  $40 \times 40$  and  $49 \times 49$  so that squares of the both types appeared. What is the minimal possible  $n$ ?