HSE Global Scholarship Competition – 2021
DEMONSTRATION COMPETITION TASK
in MATHEMATICS
10^{th} grade

Execution time – 180 minutes
Maximal mark – 100 points

1. **(7 points)** We call a rectangle *nice* if its lengths of sides are integer and the rectanlge's perimeter (in meters) numerically equals to its area (in square meters). Compute the total area of all different *nice* rectangles. *Rectangles which differ just by order of their sides, like* 10×20 *and* 20×10, *are considered same.*

2. **(7 points)** Find the total length of intervals defined on the coordinate line with: $25x^2 - 4|8 - 5x| < 80x - 64$.

3. **(7 points)** Compute the integer part of the value: $\left(\dfrac{1+\sqrt{5}}{\sqrt{2}}\right)^6 + \left(\dfrac{1-\sqrt{5}}{\sqrt{2}}\right)^6$.

4. **(7 points)** Find such value of a that the sum of squared roots of an equation $x^2 + x\sqrt{a^2 - 12a} + a - 3 = 0$ is minimal.

5. **(7 points)** Find the maximal value for xy among integer solutions (x, y) of the following system:
$$\begin{cases} 3x^2 - 8xy - y^2 = 18 \\ x^2 + y^2 - 2x + 8y + 16 = 0 \end{cases}$$

6. **(7 points)** Compute the number of integers containing no units (digit '1') in their decimal notation, each having the product of its digits equal to 300?

7. **(13 points)** 40 identical balls are rolling along a straight line. All of them have speed equal to v directed independently from each other in one of the two possible ways. When any 2 of them collide they change their direction immediately to the opposite and keep the speed v. What is the maximal possible number of collisions?

8. **(13 points)** A circle γ is inscribed in an isosceles trapezoid $ABCD$ (with bases AB and CD). Let the circle touch the side BC in a point T and let P be the second intersection point of AT and γ. Compute a ratio AB/CD if $AP/AT = 7/23$.

9. **(16 points)** For a prime number $p > 3$ there exist such positive integers k, ℓ, m and n that $p^k + p^\ell + p^m = n^2$. Prove that $p + 1$ is divisible by 8.

10. **(16 points)** All positive integer numbers with not more then 20 (decimal) digits are divided into 2 groups: those with odd sum of digits and those with even sum of digits. Prove that the sum of the 10^{th} powers of numbers in the first group equals to the sum of the 10^{th} powers of numbers in the second group.