

HSE Global Scholarship Competition – 2022  
DEMONSTRATION COMPETITION TASK  
in MATHEMATICS  
10<sup>th</sup> grade

Execution time – 180 minutes  
Maximal mark – 100 points

1. **(7 points)** We call a rectangle *nice* if its lengths of sides are integer and the rectangle's perimeter (in meters) numerically equals to its area (in square meters). Compute the total area of all different *nice* rectangles. *Rectangles which differ just by order of their sides, like  $10 \times 20$  and  $20 \times 10$ , are considered same.*
2. **(7 points)** Find the total length of intervals defined on the coordinate line with:  
 $25x^2 - 4|8 - 5x| < 80x - 64$ .
3. **(7 points)** Compute the integer part of the value:  $\left(\frac{1 + \sqrt{5}}{\sqrt{2}}\right)^6 + \left(\frac{1 - \sqrt{5}}{\sqrt{2}}\right)^6$ .
4. **(7 points)** Find such value of  $a$  that the sum of squared roots of an equation  $x^2 + x\sqrt{a^2 - 12a} + a - 3 = 0$  is minimal.
5. **(7 points)** Find the maximal value for  $xy$  among integer solutions  $(x, y)$  of the following system:
$$\begin{cases} 3x^2 - 8xy - y^2 = 18 \\ x^2 + y^2 - 2x + 8y + 16 = 0 \end{cases}$$
6. **(7 points)** Compute the number of integers containing no units (digit '1') in their decimal notation, each having the product of its digits equal to 300?
7. **(13 points)** 40 identical balls are rolling along a straight line. All of them have speed equal to  $v$  directed independently from each other in one of the two possible ways. When any 2 of them collide they change their direction immediately to the opposite and keep the speed  $v$ . What is the maximal possible number of collisions?
8. **(13 points)** A circle  $\gamma$  is inscribed in an isosceles trapezoid  $ABCD$  (with bases  $AB$  and  $CD$ ). Let the circle touch the side  $BC$  in a point  $T$  and let  $P$  be the second intersection point of  $AT$  and  $\gamma$ . Compute a ratio  $AB/CD$  if  $AP/AT = 7/23$ .
9. **(16 points)** For a prime number  $p > 3$  there exist such positive integers  $k, \ell, m$  and  $n$  that  $p^k + p^\ell + p^m = n^2$ . Prove that  $p + 1$  is divisible by 8.
10. **(16 points)** All positive integer numbers with not more than 20 (decimal) digits are divided into 2 groups: those with odd sum of digits and those with even sum of digits. Prove that the sum of the 10<sup>th</sup> powers of numbers in the first group equals to the sum of the 10<sup>th</sup> powers of numbers in the second group.