HSE Global Scholarship Competition – 2022
DEMONSTRATION COMPETITION TASK
in MATHEMATICS
11^{th} grade

Execution time – 180 minutes
Maximal mark – 100 points

1. **(7 points)** Points P and Q are the two opposite vertices of a cube with an edge length 6. Two balls of radii 1 and 2 are inside of the cube. One of them touches all the three faces of the cube containing P and another touches all the three faces containing Q. Find the distance between the balls' centers.

2. **(7 points)** Find the total length of intervals of negative numbers which satisfy the following condition:
$\frac{2\sqrt{x+3}}{x+1} \le \frac{3\sqrt{x+3}}{x+2}$.

3. **(7 points)** Find the maximal remainder of division a square trinomial $-x^2-x+13$ by a linear binomial $4x-a$ (among all real values of the parameter a).

4. **(7 points)** For an arbitrary real $x \neq 0$ and a function $f(x)$ the following condition holds: $f(\frac{x^2+49}{x}) = \frac{4x-2x^2-98}{x^2+49}$. Compute $f(16)$.

5. **(7 points)** Compute the sum $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+\ldots+[\sqrt{48}]+[\sqrt{49}]$.
By brackets we denote the lower integer part of a real number

6. **(7 points)** How many different positive integer divisors of a number $2^5 \cdot 3^3 \cdot 5^2$ have themselves an odd number of positive integer divisors?

7. **(13 points)** Let point O be a center of a circle with radius equal to 5. AB is a chord with length 6 in the circle. A square $PQRS$ is inscribed to a sector AOB so that point P belongs to a segment OA, point Q belongs to a segment OB and points R and S belong to the circle. Find the area of the square $PQRS$.

8. **(13 points)** A convex 100-gon is drawn on a checkered paper with vertices in nodes of the grid (in vertices of some unit cells). Find the maximal possible number of diagonals of the polygon which are parallel to the lines of the grid (i.e. to sides of the unit cells)?

9. **(16 points)** Find all positive integers n so that each integer written down in decimal record with $n-1$ digits '1' and one digit '7' is a prime number (for example 1711 is not because $1711 = 29 \cdot 59$).

10. **(16 points)** 2021 points on a plane are colored into two colors and some of the points are connected to each other with segments. Each minute (simultaneously) all points which are connected with even number of points change their color. Prove that initial coloring couldn't repeat after odd number of minutes.