HSE Global Scholarship Competition – 2021
Mathematics
10th grade, variant 2

1. Letters a, b, c and d denote some integer numbers from 1 to 9 in an expression $\frac{a}{b} + \frac{c}{d} = \frac{29}{30}$. Compute $a + b + c + d$.

2. Find the total length of intervals of values x from $[-10, 10]$ on the real axis which satisfy the following inequality:
$$\frac{1}{x} + \frac{2}{x+5} - \frac{3}{x+10} \geq 0$$

3. How many integer values x satisfy the inequality: $\sqrt{17-2x} \leqslant \frac{\sqrt{x^3-6x^2+12x-7}}{\sqrt{x-1}}$.

4. Solve the ineqaulity $\frac{25}{x^2+18x+82} \leq -18x - x^2 - 72$. Write down the sum of the squares of its integer solutions.

5. Evaluate the sum $\left\lfloor \frac{11}{18} \right\rfloor + \left\lfloor \frac{2 \cdot 11}{18} \right\rfloor + \left\lfloor \frac{3 \cdot 11}{18} \right\rfloor + \ldots + \left\lfloor \frac{i \cdot 11}{18} \right\rfloor + \ldots + \left\lfloor \frac{17 \cdot 11}{18} \right\rfloor$.
By $\lfloor x \rfloor$ we denote the function of (lower) integer part of a number x, i.e., by definition, it is the maximum integer number not exceeding x.

6. Four family couples (a wife and a husband) came for a dinner. They want to sit according to the following rules:
• Each person wants to sit next to his (her) spouse;
• Each person wants to have one man and one woman as his (her) neighbors.
How many different ways to seat all these people exist?
Two ways to seat the people which differ by shifting each person by the same number of positions around the table are considered different

7. A right triangle ABC is given so that $\angle B = 90°, AB = 3, BC = 6$. Let G be the centroid (medians intersection point) of the triangle. Such points A', B', C' are chosen on the sides BC, AC and AB that GA' is perpendicular to BC, GB' is perpendicular to AC and GC' is perpendicular to AB. Find the area of the triangle $A'B'C'$.

8. Consider a set $M = \{1, 2, \ldots, 50\}$. We call its subset *nice* if it contains no six consecutive numbers and has the maximum possible number of elements for this condition. Find the number of nice subsets.

9. Solve the equation in positive integer numbers $x! + y! = 5z!$.
By $n!$ we denote function "n factoreal" which is defined for positive integer numbers as follows: $n!$ equals to the product of all positive integer numbers from 1 to n. For example, $1! = 1, 2! = 1 \cdot 2 = 2, 3! = 1 \cdot 2 \cdot 3 = 6, \ldots$

10. A group of tourists consists of 50 people. There exist two unacquainted persons among any three persons of the group. It turned out that the group couldn't be seated into two buses so that there is no acquainted persons inside each of the buses. Prove there is a tourist with not more then 20 acquainted persons in the group.