HSE Global Scholarship Competition – 2021
Mathematics
10th grade, variant 3

1. Six friends: Anna, Boris, Chulpan, Dima, Evgeniy and Fyodor stand in a circle (namely in this order, according to the first letters of their names: A, B, C, D, E, F. Another person next to F is A). Each of them quietly has told his (her) favourite number to his (her) two neighbors. Than each person says loudly the sum of the two numbers the person has heard. Anna called number 38, Chulpan called number 22, Evgeniy called number 26. What is the Fyodor's favourite number?

2. We denote as α the maximum value of the expression $\frac{x+6}{x^3+216}$. Compute $\frac{1}{\alpha}$.

3. Find the minimum positive value b so that the equation $x^2+bx+12b$ has two different integer roots.

4. Consider a sqare trinomial $f(x) = ax^2+bx+c,\ a \neq 0$. Find such values a, b and c that the values $f(1),\ f(2)$ and $f(4)$ are consequtive terms of some arithmetic progression and also the equality $b^2-a^2=80$ holds. Write down the absolute value of b as the answer.

5. Determine the minimum value of the expression x^2+y^2 if x and y satisfy the condition $x^2+18x-y^2-2y=-80$.

6. Find the number of triples if integer numbers (x, y, z) which are solutions of the equations system:

$$\begin{cases} x+y+z=10 \\ |x|+|y|+|z|=16. \end{cases}$$

7. A checkered board 9×9 is given which is chess colored in two colors. We denote by N the number of arrengements of 8 chess rooks on the board so that all the rooks stand on cells of the same color (not more then one rook on a cell) and there are no two rooks standing in the same row or in the same column. Compute $N/5!$.

   A coloring of a checkered board is called chess coloring if each cell 1×1 of the board is colored entirely either in black or white so that any two cells adjacent by their common side have different colors.

   $k!$ is a notation of the factoreal of a positive integer number k. By definition $k! := 1\cdot 2\cdot 3\cdot \ldots \cdot k$.

8. A convex quadrilateral $ABCD$ has equal angles A and C, and the bisector of angle B passes through the midpoint of the side CD. Compute $\frac{BC}{AB}$ if $CD=3AD$ is given.

9. A school offeres 10 different additional classes for its students. It turned out that for each two students of the school there exists a class which is attended by one of the students and is not attended by another one. Is it possible the school has more then 1000 students?

10. A positive integer number n is such that just 25 pairs of positive integer numbers x and y exist so that $\frac{1}{x}+\frac{1}{y}=\frac{1}{n}$. Prove that n is a perfect square.