HSE Global Scholarship Competition – 2021
Mathematics
11th grade, variant 1

1. Bogdan fills up a water tank of volume 2 m^3 from a nearby river using a jar. The jar's bottom has a shape of a square with side length 30 cm and its height is equal to 40 cm. What is the minimal number of rounds to the river and back Bogdan needs to do in order to fill the tank completely?

2. Let's denote as α the maximal real value x among $x < 1$ for which the following inequality holds:
$$\frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1} \geq 0$$
Compute $\frac{1}{\alpha^2}$.

3. Find the minimum value which the sum of squares of roots of the equation can take: $x^2 + (2a+3)x + 2a$.

4. Find the maximum value of the following expression: $\frac{x^2+1}{x^4+\frac{33}{256}}$.

5. Consider a geometric progression $b_1, b_2, b_3, \ldots$ with the first term $b_1 > 0$ and the common ratio $q \neq 0$. Let q_0 be the maximum value among q for which the condition $b_3 \leq b_2 + b_1$ holds. Compute $q_0^5 + q_0^4 - 8q_0$.

6. A number is chosen at random and equally likely from 1 to 3000. Find the probability that the number is neither divisible by 3 nor by 10.

7. An angle ABC of an equilateral triangle ABC ($AB = BC$) equals to $82°$. Consider such point M inside the triangle that $AM = AB$ and $\angle MAC = 11°$. Find the value of an angle MCB. Provide the answer in degrees.

8. Let S be such set of positive integer numbers that for any x, y from S such that $x < y$ the following inequality holds: $xy < 111y - 148x$. Find the maximum number of elements which S can contain.

9. Each face of a regular tetrahedron with an edge length equal to 2020 is divided into 2020^2 equilateral triangles with side length equal to 1 (with lines parallel to the edges of the tetrahedron). Two friends Feodor and Sergey mark triangles of side length 1 alternating, one triangle in each player's turn. Feodor starts. Each marked triangle besides the first one should have at least one common point with a triangle marked by the opponent in his previous turn. The triangles could be marked just once. The player who can't make his turn loses. Who has a winning strategy and what is it?

10. Positive integer numbers a and b satisfy the condition $b^2 = a^2 + ab + b$. Prove that b is a square of an integer number.