HSE Global Scholarship Competition – 2021
Mathematics
11th grade, variant 3

1. Find the minimum 5-digit number divisible by 11 with all digits being different from each other.

2. Determine the length of the intersection of a segment $[-4, 4]$ with a set of all such values x that the following inequality holds:
$$x^2 + x + 1 \geq -\frac{1}{x}$$

3. Solve the equation $ln(x+1) \cdot \sqrt{x^3 - 8x^2 + 15x + 25} = ln((x+1)^{x+2})$.
Write down the sum of the squares of the solutions.

4. Find the maximum value of such parameter a that the equation $x + a = \sqrt{1 - x^2}$ has at least one real solution. Provide a^6 as the answer.

5. Determine the number of such pairs (x, n) of positive integer numbers that the conditions $x \geq 2$ and $x^n \leq 100$ hold.

6. Every weekday Vova does his homework listening to his favourite music. For these purposes each weekday he takes equally likely one of his 3 favourite musical discs. Let α be the probability that during the entire week he listens to each disc at least once. Compute $\frac{1}{\alpha}$.

7. For positive integer numbers $n \leq 2019$ we denote $S_n = \frac{1}{n} + \frac{1}{n+1} + \ldots + \frac{1}{2019}$. Compute the sum $S_1 + S_1^2 + S_2^2 + \ldots + S_{2019}^2$.

8. Consider a right isosceles triangle ABC with a hypotenuse $AB = \sqrt{2}$. Determine the minimum possible area of a triangle XYZ inscribed in the triangle ABC (so that X, Y, Z belong to the segments BC, CA, AB respectively) if the triangle XYZ is also isosceles and right ($\angle X = 90°$).

9. Find all such positive integer numbers m and n that a number $3^n + 1$ is divisible by $3^m - 1$.

10. Kostya colored cells 1×1 of a checkered board 100×100 into four colors so that each row and each column contains 25 cells of each color. Prove there exist such two rows and two columns that the four cells in their intersection are colored into four different colors.