

# HSE Global Scholarship Competition – 2020

## Mathematics

### 10th grade, variant 1

1. For some geometric progression  $b_1, b_2, \dots, b_n, \dots$  a value  $\frac{b_1 b_4 b_5}{(b_2)^2}$  also belongs to the progression. Find its index in the progression.
2. Solve an inequality:  $|\sqrt{x} - x| > 2|x|$ .
3. Compute  $\frac{1 + \alpha}{1 - \alpha} + \frac{1 + \beta}{1 - \beta}$  where  $\alpha$  and  $\beta$  are different roots of  $x^2 - 17x + 20$ .
4. Find the maximal value of a parameter  $a$  such that an equation  $\sqrt{x + 2a - 1} + \sqrt{x - a} = 1$  has at least one solution.
5. Solve the system:
$$\begin{cases} x + xy + y = 55 \\ x + xz + z = -15 \\ y + yz + z = -5 \end{cases}$$
6. On an Octoparty each octopus makes exactly 4 ‘legshakes’ with each other. How many ‘legshakes’ were made on the party with 37 octopi?
7. Let point  $I$  be the intersection point of angle bisectors  $AP$  and  $BQ$  of a triangle  $ABC$ . Find the length of  $PQ$  if  $\angle AIB = 120^\circ$  and  $IQ = 3$ .
8. Cells of a checkered board  $11 \times 11$  are colored white. We may pick up any 4 white cells, lying in the vertices of a square with sides parallel to the sides of the board, and recolor black any 2 diagonal cells among that 4 cells. What maximal number of black cells could appear on the board?
9. For a prime number  $p > 3$  there exist such positive integers  $k, \ell, m$  and  $n$  that  $p^k + p^\ell + p^m = n^2$ . Prove that  $p + 1$  is divisible by 8.
10. 11 vertical and 11 horizontal straight line segments are given on a plane. Prove they can't form such construction that each horizontal segment intersects 10 vertical segments and each vertical segment intersects 10 horizontal segments.