HSE Global Scholarship Competition – 2020
Mathematics
10th grade, variant 2

1. For some arithmetic progression $a_1, a_2, \dots, a_n, \dots$ a value $a_1 + 2a_4 + a_3 - 3a_2$ also belongs to the progression. Find its index in the progression.

2. Find the maximal value of an expression $13 - 2n - n^2$ among integer n.

3. Compute $\sqrt{\alpha} + \sqrt{\beta}$ where α and β are different roots of $x^2 - 31x + 81$.

4. Find all such values of a parameter a that $4a$ is an integer and a function $f(x) = |x - a| - x^2$ doesn't exeed 1.

5. Solve the system:
$$\begin{cases} x\sqrt{x} + 3y\sqrt{x} = 4,5 \\ y\sqrt{y} + 3x\sqrt{y} = 3,5 \end{cases}$$

6. Two gears in a mechanism are connected by their gear teeth so that one of the gears rotates another one. The teeth of each gear are going evenly and touching teeth of another gear just when crossing centers line of the gears. One gear has 111 teeth and another one has 481 teeth. How many full circles of the smaller gear will it take to repeat the mutual position of the teeth which are touching right now?

7. A square with a center O is built on a hypotenuse AB of a right triangle ABC so that AB is a side of the square and the square don't overlap with the triangle. Find CO if $AC = 6$ and $BC = 8$.

8. 'Shooter' is a new chess figure which attacks at any distance but just in one direction horizontally or vertically (for example, attacking just horizontally to the left from itself). The figure can't attack through another figure on a checkered board. What is the maximal number of 'shooters' none of them attacking others could stand on a checkered board 20×20?

9. Integers from 1 to 600 are written down in one line in such order that after crossing out all numbers bigger then 300 the rest form an increasing sequence. Also after crossing out all numbers not exeeding 300 the rest form a decreasing sequence. Prove that the total of numbers on positions starting from 151-st till 450-th is divisible by 3.

10. Twenty-five men sit around a circular table. Every hour they vote, and everyone must respond 'yes' or 'no'. They decide how to answer on the $(n+1)$-th voting according to answers on the n-th voting:

    • if a man responded on n-th voting the same as at least one of his two neighbours, then he gives the same response on the $(n+1)$-th voting;

    • if his response differs from both neighbours' responses on the n-th voting, then he changes his response on the $(n+1)$-th voting.

    Prove that regardless particular results of the first voting the people will stop changing their responses at some moment.