

HSE Global Scholarship Competition – 2020

Mathematics

10th grade, variant 3

1. We call a rectangle *nice* if its lengths of sides are integer and the rectangle's perimeter (in meters) numerically equals to its area (in square meters). Compute the total area of all different *nice* rectangles. *Rectangles which differ just by order of their sides, like 10×20 and 20×10 , are considered same.*
2. Find the total length of intervals defined on the coordinate line with: $25x^2 - 4|8 - 5x| < 80x - 64$.
3. Compute the integer part of the value: $\left(\frac{1 + \sqrt{5}}{\sqrt{2}}\right)^6 + \left(\frac{1 - \sqrt{5}}{\sqrt{2}}\right)^6$.
4. Find such value of a that the sum of squared roots of an equation $x^2 + x\sqrt{a^2 - 12a} + a - 3 = 0$ is minimal.
5. Find the maximal value for xy among integer solutions (x, y) of the following system:

$$\begin{cases} 3x^2 - 8xy - y^2 = 18 \\ x^2 + y^2 - 2x + 8y + 16 = 0 \end{cases}$$

6. Compute the number of integers with product of (decimal) digits equal to 300?
7. 40 identical balls are rolling along a straight line. They all have speed equal to v , but some of them might move in opposite directions. When some 2 of them collide they immediately change their direction to the opposite and keep the speed v . What is the maximal number of collisions which can happen?
8. A circle γ is inscribed in an isosceles trapezoid $ABCD$ (with bases AB and CD). Let the circle touch the side BC in a point T and let P be the second intersection point of AT and γ . Compute a ratio AB/CD if $AP/AT = 7/23$.
9. Find all positive integer solutions of an equation $n^{n-1} = 4m^2 + 2m + 3$.
10. All positive integer numbers with not more than 20 (decimal) digits are divided into 2 groups: those with odd sum of digits and those with even sum of digits. Prove that sum of the 10^{th} powers of numbers in the first group equals to sum of the 10^{th} powers of numbers in the second group.