

# HSE Global Scholarship Competition – 2020

## Mathematics

### 11th grade, variant 1

1. Determine all numbers  $a$  such that  $\frac{1}{2}, a, a^2$  form an arithmetic progression (in the mentioned order).
2. Solve an equation:  $\sqrt{x^2 - 5x + 12} + x^2 = 5x$ .
3. Roots of an equation  $4x^2 - ax + 1 = 0$  vary on a real axis while a real parameter  $a$  varies from 4 to 5. Find the total length of intervals which the roots traveled on the axis.
4. For a given function  $f$  and any real  $x \neq 3$  the following equality holds:  $f\left(\frac{1}{3-x}\right) = 5x + 1$ . Determine the total of all such positive integers  $y$  that  $f(y)$  is integer.
5. Determine all possible  $a$  such that an equation  $x^2 + (a - 2)^2 = |x - 2 + a| + |x - a + 2|$  has exactly one solution.
6. An intellectuals' club has 30 participants, each having a personal number ranged from 1 to 30 (all personal numbers are integer and different from each other). The club may apply a team of arbitrary size for a competition. Yet it is forbidden to have team members with one's personal number being twice bigger than another's personal number. What is the maximal size of a team which the club may apply for the competition?
7. An inheritance consists of several diamonds with total cost \$1000 000. It is possible both to divide it into 5 equal parts and to divide it into 8 equal parts. What is the maximal possible cost for the smallest diamond in the inheritance?
8. An angle  $A$  of a rhombus  $ABCD$  is equal to  $120^\circ$ . A point  $M$  inside the rhombus is such that  $AM = 1, CM = 2$  and  $BM = 3$ . Determine  $DM$  and  $AB$ .
9. Distinct positive integer numbers  $a, b$  and  $c$  are such that  $b + c + bc$  is divisible by  $a, c + a + ca$  is divisible by  $b$  and  $a + b + ab$  is divisible by  $c$ . Prove at least one of the numbers  $a, b$  and  $c$  is composite.
10. 1010 not intersecting segments with ends in vertices of a right 2020-gon divide the vertices into pairs. Prove that directions could be assigned to the segments in such way that the total of the obtained vectors is equal to zero.