

# HSE Global Scholarship Competition – 2020

## Mathematics

### 11th grade, variant 2

1. Determine all possible values for  $x - y$  if  $x + y = 6$  and  $xy = 2$ .
2. Find the maximal negative solution of an equation  $\cos\left(\frac{\pi(x-1)}{3}\right) = \frac{1}{2}$ .
3. Find the total length of intervals on the coordinate axis defined by a condition  $|5x - 3| < \sqrt{x}$ .
4. Function  $f(x) = e^{-x} - 2x$  is given. Solve the equation  $f(4x) = f(1 - x^2)$ .
5. Find all solutions of an equation  $\sqrt{x} = \sqrt{[x]} + \sqrt{\{x\}}$  which belong to an interval  $(3, 6)$ .
6. How many sides of unit cells does intersect a diagonal of a checkered rectangle  $30 \times 5$  if passing through a cell's vertex doesn't count as an intersection?
7. A convex 100-gon is drawn on a checkered paper with vertices in nodes of the grid (in vertices of some unit cells). Find the maximal possible number of diagonals of the polygon parallel to lines of the grid (i.e. to sides of the unit cells)?
8. A point  $M$  inside an isosceles right triangle  $ABC$  ( $\angle C = 90^\circ$ ) is such that  $AM = 2$ ,  $\angle AMB = 120^\circ$ ,  $\angle AMC = 105^\circ$ . Find  $BM$  and  $CM$ .
9. Find all such positive integer numbers  $n$  that positive integers  $a$ ,  $b$ ,  $c$  and  $d$  exist so that  $a + b + c + d = 2n\sqrt{abcd}$ ?
10. 2021 points on a plane are colored into two colors and some of the points are connected to each other with segments. Each minute (simultaneously) all points which are connected with even number of points change their color. Prove that initial coloring couldn't repeat after odd number of minutes.