HSE International Olympiad – 2023

MATHEMATICS

10th grade

Demonstration Test

Time allowed - 180 min
Maximum grade - 100 points

1. *Just answers are expected for problems of the* <u>*first block №№ 1-6*</u>*. You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the* <u>*second block №№ 7-8*</u> *should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the* <u>*third block №№ 9-10*</u> *are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

Compute $\dfrac{0,6\cdot(1,8\cdot 0,5)\cdot 10}{(0,25\cdot 8\cdot 4)\cdot 0,375}$. (7 points)

Answer: 1,8

Problem 2.

A train ticket from Perm to Kazan costs 750 rubles for a person. A car consumes 6 liters of fuel per 100 kilometers of a ride. Road distance for a car between the cities is 800 kilometers and the fuel price is 26 rubles per liter. Determine the cost (in rubles) of the cheapest (one way) trip from Perm to Kazan for 2 persons. (7 points)

Answer: 1248

Problem 3.

Compute the product of the roots of an equation $(x^2+2)^2+x^4=20$. (7 points)

Answer: -2

Problem 4.

Compute the sum of all such integer values x that the following function is defined:

$$y=\sqrt{|x-3|\cdot(x-2)}+\sqrt{3-|x-1|}+\sqrt{x^2+2x-15}$$

(7 points)

Answer: 7

Problem 5.

A circle is inscribed into an isosceles triangle. The tangency point of the circle with the triangle's lateral side divides the side into segments of length 2 and 3 cm counting from the mutual vertex of the equal sides. Compute the area of the triangle (in cm^2). (7 points)

Answer: 12

Problem 6.

Compute the minimum integer value a such that one of the roots of an equation $x^2 + 3x - a - 5 = 0$ is greater than 2 and another one is less than 2. (7 points)

Answer: 6

Problem 7.

Points B and C belong to a circle with a diameter AD, l is a tangent line to the circle which contains the point D. We denote as P the intersection point of lines AB and l and as Q we denote the intersection point of lines AC and l. Compute the length of CQ provided $AB = 46.08, AC = 28.8$ and $BP = 3.92$.

Answer: 51,2 (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

Determine all such real values x that equality $[50x + 97] = 50 + 97x$ holds.

Remark: by square brackets $[x]$ we denote a function of taking a (lower) integer part of a real number x (that is the maximum integer number not exceeding x).

Answer: 95/97; 96/97; 1 (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

Two players play the following game on a checkered board 7×7. The first player chooses a unit square of the board to put a knight (chess figure) there. Then players move the figure one by one (starting with a move of the second player) under chess rules. Stepping outside the board or getting into already visited unit square is forbidden. The player who can not make a move on his turn loses. Who has a winning strategy in?

Remark: a knight moves like following: first it moves by 2 squares horizontally of vertically from the current position and then it moves by 1 more square in a perpendicular direction. The step is considered to be done as one 'jump' from the initial square of the move into the final square of the move. (16 points)

In this problem you are expected to present a **full solution:**

Answer: The first player can ensure their win.

Problem 10.

Find all such prime numbers p that there exist such integer numbers m and n that $p = m^2 + n^2$ and a number $m^3 + n^3 + 8mn$ is divisible by p. (16 points)

In this problem you are expected to present a **full solution:**

Answer: 2; 5; 13.