HSE International Olympiad – 2023

MATHEMATICS

11th grade

Demonstration Test

Time allowed - 180 min
Maximum grade - 100 points

1. *Just answers are expected for problems of the* *first block №№ 1-6*. *You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the* *second block №№ 7-8* *should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the* *third block №№ 9-10* *are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

Evaluate $x + y$ provided $x^2 + 4x + y^2 - 6y + 13 = 0$.

Answer: 1 (7 points)

Problem 2.

Some numbers a, b, c satisfy a condition $a : b : c = 2 : 3 : 15$. The number a decreased by 10%, b increased by 20% and number c remained same. By how many percent have their sum changed?

Answer: 2 (7 points)

Problem 3.

Compute the number of integers lying between the two roots of the following equation:

$$2x^2 + 3x - 17 = 2(11 - 4\sqrt{7}) + 3(\sqrt{7} - 2) - 17$$

Answer: 3 (7 points)

Problem 4.

Determine the minimum integer value x such that the following function is defined:

$$y = \sqrt{\frac{3 + x}{x - 1}} + \sqrt{x}$$

Answer: 2 (7 points)

Problem 5.

The area of an equilateral triangle is equal to $16\sqrt{3}/3$ cm^2. Compute the length of its bisector (in cm).

Answer: 4 (7 points)

Problem 6.

Determine the maximum positive integer a such that the following system of equations has less than four solutions: $\begin{cases} \sin(x+y)=0 \\ x^2+y^2=a^2 \end{cases}$

Answer: 2 (7 points)

Problem 7.

Find all such positive real numbers x that

$$\frac{1}{[x]} - \frac{1}{[2x]} = \frac{1}{6\{x\}}$$

Remark: by square brackets $[x]$ we denote a function of taking a (lower) integer part of a real number x (that is the maximum integer number not exceeding x). By braces $\{x\}$ we denote the fractional part of a real number x, which is by definition $\{x\} := x - [x]$.

Answer: 4/3; 23/9; 31/8 (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

Angle B of a triangle ABC equals to $60°$ and angle C of the triangle equals to $54°$. Point P is denoted on a side BC such that the perimeter of a quadrilateral $ACPM$ equals to the perimeter of a triangle PMB *where point M is the center of AB.* Determine the value of an angle MPB.

Answer: 27° (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

71 first graders run in a gymnasium class. Two high school students Nick and Serg received a list of names of these first graders and got a task to put the names in an accending order of height of the first grade students (it is known that all of them have different height). Nich and Serg decided to do the following: Nick reads names of some 3 studentsfrom the list and Serg catches them and tells to Nick who of the three has the intermediate height. Nick can not see that by himself, he just hears Serg's words. What maximum number of the first grade studentscan Nick put correctly (according to height) to a reordered list after 1225 answers of Serg? (16 points)

In this problem you are expected to present a **full solution:**

Answer: Based on the given number of quetstions it is possible only to determine the position of one (middle by height) student.

Problem 10.

Positive integer numbers m and n are such that $3^m - 2^n$ is divisible by 47. What could be the remainder of the division of $4m + n$ by 23? (16 points)

In this problem you are expected to present a **full solution:**

Answer: 0