To be completed by the Jury. Please don't make any notes here!											
CODE	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	**Total points**
	Max 7	Max 7	Max 7	Max 7	Max 7	Max 7	Max 13	Max 13	Max 16	Max 16	**Max 100**

MATHEMATICS

10th grade

Variant 1

Time allowed - 180 min
Maximum grade - 100 points

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1. *Just answers are expected for problems of the first block №№ 1-6. You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the second block №№ 7-8 should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the third block №№ 9-10 are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

Consider an operation which produces result $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$ for any three nonzero numbers x, y and z. We denote it as $G(x, y, z)$. Evaluate $G(2, 12, 9)$.

Answer: ______________ (7 points)

Problem 2.

Tamara has 5 daughters and no sons. Some of her daughters have 5 daughters, and the rest have none. Tamara has a total of 20 daughters and granddaughters, and no great-granddaughters. How many of Tamara's daughters and grand-daughters have no daughters?

Answer: ______________ (7 points)

Problem 3.

Paul is using cans of condensed milk to build a fortress shaped as elongated pyramid. The ground level of his fortress is a rectangle 8 cans by 12 cans. Each next level is also a rectangle of cans where each can is placed on top of the can of the lower layer. The size of this rectangle is such that the lower level contains a can-free rim that is one milk can wide. Compute how many cans are required to build the entire fortress.

Answer: ____________ (7 points)

Problem 4.

Compute the sum of all digits in the number $111\,111\,111^2$.

Answer: ______________ (7 points)

Problem 5.

A sequence $a_1, a_2, \ldots$ is defined with a recurrent formula $a_{n+2} = 3a_{n+1} - a_n$ (which holds for each positive integer value of n). Compute a_5 provided that $a_6 = 343$ and $a_3 = 19$.

Answer: ________________ (7 points)

Problem 6.

The central unit square in a 7×7 grid is black, all other unit squares of the grid are white. How many squares (of all possible sizes) with borders matching the grid lines contain the black square?

Answer: ______________ (7 points)

Problem 7.

A circle is given with a diameter AB. Let point C divide this diameter in a way that $AC : BC = 1 : 2$. Let P and Q be points on the circle such that $PC \perp AB$ and PQ is another diameter. What is the ratio of the area of triangle PCQ to the area of triangle ABP?

Answer: ______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

Worker ants are moving along a straight line in one column at the same constant speed. The distance between the first and the last ant in the column is 15 meters. A guard ant passes through the entire column starting from the last ant and on reaching the first ant turns back and passes through the column down to the last worker ant again. The point where the guard and the last worker ant meet is precisely 8 meters away from the point where the guard ant started its journey. Determine the total distance covered by the guard.

Answer: ______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

Consider all nine-digit positive integers which contain only digits $1, 2$, and 3 (the smallest number in the range is 111111111 and the biggest one is 333333333). Each number was written down on a new card, so all cards make up a deck of 19683 cards. Fred, Sam and Tim deal the cards in line with a rule: any two cards given to the same boy must contain at least one matching digit in a matching position. It turned out that Fred holds a card with number 133221311 and Sam holds a card with number 133211311. Determine which of the three boys holds a card with number 123123123.

(16 points)

In this problem you are expected to present a **full solution:**

Problem 10.

Determine all positive integers n which have a positive integer divisor d such that $n^4 + d^3$ is divisible by $n^2 d + 1$.

(16 points)

In this problem you are expected to present a **full solution:**