To be completed by the Jury. Please don't make any notes here!											
CODE	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	**Total points**
	Max 7	Max 7	Max 7	Max 7	Max 7	Max 7	Max 13	Max 13	Max 16	Max 16	**Max 100**

MATHEMATICS

10th grade

Variant 2

Time allowed - 180 min
Maximum grade - 100 points

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1. *Just answers are expected for problems of the* *<u>first block №№ 1-6</u>*. *You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the* *<u>second block №№ 7-8</u>* *should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the* *<u>third block №№ 9-10</u>* *are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

A bakery owner turns on his doughnut machine at 7:30 AM. By 9:40 AM the machine has completed one third of the day's job. At what time will the doughnut machine complete the etire day's job? *In your answer use military time (24 hours) with a decimal point instead of a colon. For example, 1:31 PM should be written as* 13.31.

Answer: ______________ (7 points)

Problem 2.

The Night Soccer League players want to buy socks and T-shirts. Socks cost 190 rubles per pair, and one T-shirt costs 170 rubles more than a pair of socks. Each player needs one pair of socks and a T-shirt for home games and another pair of socks and a T-shirt for away games. If the total cost is 102300 rubles, how many players are in the league?

Answer: ________________ (7 points)

Problem 3.

A sculptor purchased a rectangular piece of marble with dimensions 15 cm by 10 cm by 8 cm. He decided to make a relatively easy sculpture (by his standards) by removing cubes 3 cm by 3 cm by 3 cm from each corner of the marble piece. What percent of the original volume should be removed?

Answer: ________________ (7 points)

Problem 4.

At a summer camp 42% of children do math, 51% do sports, and 65% of math students do sports. What percent of those who don't do sports do math?

Answer: ______________ (7 points)

Problem 5.

Suppose $\sin a + \sin b = \sqrt{\frac{6}{5}}$ and $\cos a + \cos b = 1$. Find $\cos(a - b)$.

Answer: ________________ (7 points)

Problem 6.

Seven points are chosen on a circle. All chords are drawn which connect some pairs of the chosen points. It turned out that no three chords intersect in a single point. How many triangles are drawn on the picture with all three vertices inside of the circle?

Answer: ______________ (7 points)

Problem 7.

Point O is marked inside an equilateral triangle PQR. Points X, Y, and Z are the bases of perpendiculars constructed from point O to the sides PQ, QR, RP respectively. It turned out that $OX = 1, OY = 2$ and $OZ = 3$. Compute the side length of the triangle.

Answer: ______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

One hundred numbers are written in a circle. The sum of all numbers is equal to 100, and the sum of any six consecutive numbers (along the circle) is less or equal to 6. One of the numbers is equal to 6. Restore all written numbers.

Answer: _______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

Frank and Sam are playing the following game. First, Frank adds + and − signes in front of each of the fifty numbers $1, 2, \ldots, 50$ written on a blackboard. Next, the players alternate their steps one by one, starting from Sam. A step includes choosing a number (with the written sign) from the blackboard, erasing it, and writing it down in the person's notebook. Once all numbers are removed from the blackboard, each player computes the sum in his notebook and takes its absolute value. The player with the greater final number wins. Is it possible to guarantee victory for one of the players, regardless of the opponent's play? If yes, show how to do that, if no, explain why this cannot be done.

(16 points)

In this problem you are expected to present a **full solution:**

Problem 10.

Find all pairs of positive integers $m > 1$ and $n > 1$ such that $m + 1$ is divisible by n and $n^2 - n + 1$ is divisible by m.

(16 points)

In this problem you are expected to present a **full solution:**