HSE International Olympiad – 2023

To be completed by the Jury. Please don't make any notes here!											
CODE	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	**Total points**
	Max 7	Max 7	Max 7	Max 7	Max 7	Max 7	Max 13	Max 13	Max 16	Max 16	**Max 100**

MATHEMATICS

10th grade

Variant 3

Time allowed - 180 min
Maximum grade - 100 points

1. *Just answers are expected for problems of the first block №№ 1-6. You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the second block №№ 7-8 should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the third block №№ 9-10 are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

You and nine friends need to raise 18000 rubles in donations for a charity, dividing the fundraising equally. How many rubles will each of you need to raise?

Answer: ______________ (7 points)

Problem 2.

Two teams, the Wolves and the Panthers, met to play a handball game. The two teams scored a total of 36 points, and the Wolves won by a margin of 12 points. How many points did the Panthers score?

Answer: ______________ (7 points)

Problem 3.

Paul has 1000 rubles in his investment account. Each month 2% of this amount is accrued to his spending debit card. 1.45% of any amount credited to his debit card is automatically sent to his piggy bank account. How many kopecks (one hundredth part of one ruble) are transferred to Paul's piggy bank account monthly from the 1000 rubles in his investment account?

Answer: ______________ (7 points)

Problem 4.

Let $a, b, c, d,$ and e be distinct integers such that $(7-a)(7-b)(7-c)(7-d)(7-e) = 45$. Compute $a+b+c+d+e$.

Answer: ______________ (7 points)

Problem 5.

Paul and Vicky start running in opposite directions on a circular track from diametrically opposed points. They first meet after Paul has covered 120 meters. They next meet after Vicky has covered 200 meters beyond their first meeting point. Determine the length of the track in meters provided that both runners are moving at a constant speed.

Answer: ________________ (7 points)

Problem 6.

A twelve-sided regular polygon $A_1A_2\dots A_{11}A_{12}$ is given. Determine how many distinct equilateral triangles in the plane of the figure have at least two common vertices with the polygon?

Answer: ____________ (7 points)

Problem 7.

Let X be a point on the side PQ of a rhombus $PQRS$. Suppose Y and Z are the bases of perpendiculars constructed from X to diagonals PR and QS respectively. Find the minimum possible length of a segment YZ provided $PR = 10$ and $QS = 24$.

Answer: _______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

First, all integers from 1 to 2023, inclusive, without repetitions are written down in a line on a blackboard in some order. Each minute the line of numbers is changed as follows: the first and second numbers (a and b) in the line are erased and instead the absolute value of their difference ($|a-b|$) is written at the beginning of the line. No other numbers in the line are changed at this step. After 2022 steps only one number is left on the board. Find all possible options of the remaining number (for different starting order of the numbers).

Note: If the initial order is $3, 108, 11, 1021, 1, \ldots$, the line of numbers begins with $105, 11, 1021, 1, \ldots$ after the first step, after the second step it is $94, 1021, 1, \ldots$ and so on.

Answer: ______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

Two hundred small balls of the same color and size are mixed up in a bag. One hundred of them weigh 20 grams each, and the other hundred weigh 21 grams each. For performing a trick you need two groups of balls with the same quantity but different total weights. Determine the minimum number of weighings needed to form two such groups provided that you only have one pan scale (which shows either which of the two bowls weights more or shows that both wights are equal) and explain how this can be done.

(16 points)

In this problem you are expected to present a **full solution:**

Problem 10.

Three different positive integers a, b, and c are given so that $p = ab + bc + ac$ is a prime number. Prove that numbers a^3, b^3 and c^3 have different remainders after integer division by p.

(16 points)

In this problem you are expected to present a **full solution:**