HSE International Olympiad – 2023

							To be completed by the Jury. Please don't make any notes here!				
C O D E	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	**Total points**
	Max 7	Max 7	Max 7	Max 7	Max 7	Max 7	Max 13	Max 13	Max 16	Max 16	**Max 100**

MATHEMATICS

11th grade

Variant 1

Time allowed - 180 min
Maximum grade - 100 points

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1. *Just answers are expected for problems of the* *first block №№ 1-6*. *You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the* *second block №№ 7-8* *should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the* *third block №№ 9-10* *are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

Find a positive integer n such that an expression $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{n}$ is also integer.

Answer: ______________ (7 points)

Problem 2.

Determine the nearest integer to the value $\dfrac{10^{2022}+10^{2024}}{10^{2023}+10^{2023}}$.

Answer: _______________ (7 points)

Problem 3.

A grocer stacks oranges up in a pyramid-like stack whose rectangular base is 5 oranges by 7 oranges. Each orange above the first level rests in a pocket formed by four oranges below. How many oranges does the stack contain if it has no more space for another orange?

Answer: ______________ (7 points)

Problem 4.

How many positive integers n satisfy the following condition: $(130n)^{50} > n^{100} > 2^{200}$?

Answer: ______________ (7 points)

Problem 5.

The polynomial $x^3 - ax^2 + bx - 2022$ has three positive integer roots. What is the smallest possible value of a?

Answer: ________________ (7 points)

Problem 6.

A robot learns how to play tic-tac-toe: it randomly selects a unit square (equally likely) on a checkerboard 3×3 and puts a cross there. Next, it randomly puts a cross (equally likely) to one of the remaining unit squares. And finally, it does the same for the third time with the remaining unit squares. Compute the probability that these three crosses stand in one line (in one row, column or one of the two main diagonals). Give number $1/\alpha$ as your answer where α is the desired probability.

Answer: ______________ (7 points)

Problem 7.

A circle Γ with radius equal to 36 touches internally three other circles Ω, ω_1, and ω_2 which touch each other externally. It is known that Ω contains the center of the circle Γ, and the circles ω_1 and ω_2 have equal radii. Compute the length of radius of ω_1.

Answer: ________________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

Four cars A, B, C, and D travelled at a constant speed along the same highway: A, B, and C were moving in one direction and D was moving in the opposite direction. Car A passed B and C at $8:00$ and $9:00$ respectively and met D at $10:00$. Car D met B and C at $12:00$ and $14:00$ respectively. Find out when car B overtook car C.

Answer: ________________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

A checkerboard 5×5 and an unlimited supply of dominoes sized 2×1 are given, each domino having the same size as any two neighboring unit squares of the board. The dominoes can be placed on the board along the grid lines only and just in one layer. The board is called *filled up* if no more dominoes can be added to those that are already on the board. What is the minimum number of dominoes on a filled up board?

(16 points)

In this problem you are expected to present a **full solution:**

Problem 10.

Find all pairs of positive integers a and b such that $b^2 + a$ is a power of a prime number and $a^2 + b$ is divisible by $b^2 + a$.

(16 points)

In this problem you are expected to present a **full solution:**