To be completed by the Jury. Please don't make any notes here!											
CODE	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	**Total points**
	Max 7	Max 7	Max 7	Max 7	Max 7	Max 7	Max 13	Max 13	Max 16	Max 16	**Max 100**

MATHEMATICS

11th grade

Variant 2

Time allowed - 180 min
Maximum grade - 100 points

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1. *Just answers are expected for problems of the* *__first block №№ 1-6__*. *You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the* *__second block №№ 7-8__* *should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the* *__third block №№ 9-10__* *are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

Consider an operation which produces result $\frac{a}{a-b}$ for any two unequal numbers a and b. We denote it as $f(a, b)$. Evaluate $f(f(1, 3), f(3, 1))$.

Answer: ______________ (7 points)

Problem 2.

Compute the expression $(-1)^1 + (-1)^2 + \ldots + (-1)^{2023}$?

Answer: ____________ (7 points)

Problem 3.

A wooden cube with side length equal to n is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the resulting unit cubes are red. What is n?

Answer: ________________ (7 points)

Problem 4.

There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x. What is the sum of those values of a?

Answer: ______________ (7 points)

Problem 5.

Let a, b, c, d, and e be five consecutive terms in a geometric sequence such that $a \cdot b \cdot c \cdot d \cdot e = 1024$. One of the numbers a, b, c, d, and e can be evaluated uniquely by these conditions. What is this value?

Answer: ______________ (7 points)

Problem 6.

Ann and Pete are playing a game where Ann rolls three dices 1 to 8 (all the numbers are equiprobable for each dice) and computes the sum of the three rolled numbers. Pete computes the product of the rolled numbers. Ann wins if her number is not less than Pete's number. Find the probability that Ann wins. Give $1024 \times \alpha$ as the answer where α is the desired probability.

Answer: ________________ (7 points)

Problem 7.

The radius of the inscribed circle ω of an isosceles triangle ABC $(AB = BC)$ is equal to 2. Another circle is inscribed into the angle B of the triangle. It touches ω and has the radius equal to 1. Compute the area of the triangle ABC.

Answer: ______________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

A computer program generates a sequence of integers in the following way: the first number is input by Kurt and saved by the program. Next, the program performs integer division of the stored number by 18 with a remainder. The sum of the incomplete quotient and the remainder is saved instead of the previously stored number. For example if Kurt inputs 5291 the program computes: $5291 = 293 \cdot 18 + 17$ and stores $310 = 293 + 17$ instead of 5291. The next stored number is 21 as $310 = 17 \cdot 18 + 4$ and $17 + 4 = 21$, and so forth. Show that the stored value starts repeating itself infinitely for any initial number. Find this repeating value provided that the Kurt's number is 2^{2023}.

Answer: ____________ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

A player writes down a 3-digit number on a lottery ticket where each digit should be equal to $1, 2, 3$, or 4 (the digits may repeat). The lottery host announces a 3-digit number of the same type. A ticket wins if the number on it matches the announced number in at least 2 positions. The player wants to buy and fill in several tickets to make sure that at least one of them wins. Determine the minimum number of tickets the player should buy and the way they should be filled in.
Note: If the announced number is 423, the ticket with number 123 will be a winning one and the ticket with number 243 will not.

(16 points)

In this problem you are expected to present a **full solution:**

Problem 10.

Determine all prime numbers p and q and all such positive integer numbers $n > 1$ such that numbers $p^n q + 1$ and $pq^n + 1$ both are perfect squares.

(16 points)

In this problem you are expected to present a **full solution:**