HSE International Olympiad – 2023

To be completed by the Jury. Please don't make any notes here!											
CODE	Task 1	Task 2	Task 3	Task 4	Task 5	Task 6	Task 7	Task 8	Task 9	Task 10	**Total points**
	Max 7	Max 7	Max 7	Max 7	Max 7	Max 7	Max 13	Max 13	Max 16	Max 16	**Max 100**

MATHEMATICS

11th grade

Variant 3

Time allowed - 180 min
Maximum grade - 100 points

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1. *Just answers are expected for problems of the first block №№ 1-6. You may use blank space after the tasks for your notes. No other notes besides the answer will affect your mark.*

2. *Solutions for problems of the second block №№ 7-8 should contain your answer and detailed scheme of your solution with all key statements and key proof steps listed.*

3. *Full solutions for problems of the third block №№ 9-10 are expected: an answer and detailed full proof. Solutions containing just answer without proof would be considered as incomplete (or absent) and the problem would be considered unsolved.*

Problem 1.

Suppose $\frac{2}{3}$ of 10 bananas are worth as much as 8 oranges. How many oranges are worth as much as $\frac{1}{2}$ of 15 bananas?

Answer: _____ (7 points)

Problem 2.

Compute the product

$$\frac{8}{4} \cdot \frac{12}{8} \cdot \frac{16}{12} \cdot \ldots \cdot \frac{4n+4}{4n} \cdot \ldots \cdot \frac{2024}{2020}.$$

Answer: _____ (7 points)

Problem 3.

What is the difference between the sum of the first 2023 even numbers and the sum of the first 2023 odd numbers?

Answer: _____ (7 points)

Problem 4.

Find the value of x that satisfies the equation $25^{-2} = \dfrac{5^{48/x}}{5^{26/x} \cdot 25^{19/x}}$.

Answer: _____ (7 points)

Problem 5.

The first three terms of a sequence are $2023, 2024$ and 2025. Subsequent terms are constructed as follows: each three consecutive terms a_i, a_{i+1} and a_{i+2} define the next term a_{i+3} as $a_i + a_{i+1} - a_{i+2}$. For example, the fourth term is $2023+2024-2025 = 2022$. What is the 2022^{th} term in this sequence?

Answer: _____ (7 points)

Problem 6.

At the push of a button a slot machine gives out an integer ranging from 10 to 99 (equiprobably). A player wins if the decimal notation of the obtained number contains digit 3. Determine the probability of winning the game after pushing the button just once.

Answer: _____ (7 points)

Problem 7.

Consider a square $ABCD$. Suppose a circle with center A and radius AB intersects in points D and E another circle that has segment DC as its diameter. Compute the distance between point E and side AD provided that the side length of the square is equal to 6.

Answer: _____ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 8.

A 10-digit number is selected at random for a mathematical lottery. The winning numbers are all 10-digit numbers that both match the selected number in precisely 9 positions and are divisible by 7. How many winning numbers exist if the selected number is 1234567890?

Answer: _____ (13 points)

In this problem you are expected to present also a scheme of your solution (thesis proof) along with the answer. Thesis proof is a list of all important steps and key statements of a proof written down without technical details.

Thesis proof:

Problem 9.

A big rectangular carpet of unknown size is unfolded on the floor of a secret Carpet Room in a palace. It is known that the carpet's side lengths are measured in meters and have integer values. We also know that the big carpet can accomodate without overlaps 234 small 3×3 meter rectangular carpets with sides parallel to the sides of the big carpet. Determine the maximum number of 1×5 meter carpets with sides parallel to the sides of the big carpet which will definitely fit into the big carpet without overlaps.

(16 points)

In this problem you are expected to present a **full solution:**

Problem 10.

Determine all pairs of positive integers m and n such that a number $(m^2+n)(n^2+m)$ is the fifth power of a prime number.

(16 points)

In this problem you are expected to present a **full solution:**